Industrial Specialization, Financial Integration and International Consumption Risk Sharing*

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Abstract

This paper introduces a two-country, two-good model that relates financial openness, industrial specialization and consumption risk sharing. Financial liberalization provides extra consumption insurance opportunities, which in turn allows countries to take extra risks by focusing on higher risk higher return projects. The model has two sectors with linear technology and stochastic productivity shocks. Production uncertainty in the model ensures incomplete specialization in production.

This model also has implications for consumption smoothing. Consumption smoothing will depend on both the actual level of financial openness and on the productivity shock correlations between sectors and between countries. Thus, substantial liberalization does not necessarily mean an improvement in consumption risk sharing and equilibrium portfolios can be biased towards domestic assets.

1 Introduction

Standard models of international asset trade lack mechanisms linking an economy's financial openness and industrial specialization. The contribution of this paper is a simple model of global portfolio diversification in which a link between financial liberalization and specialization emerges very naturally. Within that model, an economy that liberalizes its financial markets is able to share consumption risks, which in turn would allow the country to take extra risks by specializing in its most efficient sector. Output volatility will increase and cross-country output correlations will decline.

The basic theme of this paper goes back to Arrow's (1971 p.137) observation that "*the mere trading of risks, taken as given, is only part of the story and in many respects the less interesting part. The possibility of shifting risks, of insurance in the broadest sense, permits individuals to engage in risky activities that they would not otherwise undertake.*". Obstfeld (1994) and Hnatkovska and Evans (2007) show theoretically that the insurance provided by international markets allows countries to expand production in risky sectors, thereby increasing welfare and also output volatility. The much simpler framework this paper chooses emphasizes a link between financial liberalization and specialization, which to the best of my knowledge is missing in the theoretical literature up to date. Kalemli-Ozcan, Sorensen and Yosha (2003) show that risk sharing enhances specialization in production and Kalemli-Ozcan, Papaioannou and Peydro (2009) provides empirical evidence that

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a higher degree of financial integration is associated with less synchronized output cycles. This paper rationalizes on these findings and shows how financial integration can enhance specialization and trade.

The model is a two-country, two-sector model with linear technology and stochastic productivity shocks. Production uncertainty in the model ensures incomplete specialization. When financial autarky exists, firms will allocate capital to both sectors to ensure against uncertainty in production, even though one of the sectors is more productive. As consumers engage in financial transactions and they ensure part of their consumption, countries can devote a larger share of their capital to the sector they have a comparative advantage in. As the country moves towards more liberalization this pattern of specialization increases, but it does not have to reach full specialization. The reason why there is no full specialization is because the role of international financial markets is to *share* the risks efficiently, not *eliminate* them. Even in the case of perfect risk sharing, there still exist some risks in production, which will make the consumer not fully specialize, so it can ensure internally against any remaining shocks.

A related study is Koren (2003) that tries to explain the low level of international trade (compared to the predictions of the gravity model) in a multi-sector framework with productivity shocks. In this paper, financial markets are modeled differently. Our specification allows us to observe non full specialization even if the markets are complete. While Koren (2003) requires very strict assumptions, like no country-specific shocks, this study can distinguish between country-specific and sector-specific shock correlations and can track what happens to portfolio choice and different measures of consumption smoothing for different levels of shock correlations. Also, modelling financial markets as in this study, allows us to map financial openness on portfolio choice and production specialization.

A direct consequence of the model is the effect of financial liberalization on consumption smoothing. In this model substantial liberalization may not mean an improvement in consumption smoothing, consistent with the empirical literature in this area. The actual impediments of trading foreign capital and cross-country productivity correlations can explain why empirical studies fail to find improvements in consumption smoothing as countries have become more financially liberalized. Kose, Otrok and Whiteman (2003) provide evidence of a world business cycle, which suggests a high correlation of business cycles between countries. Heathcote and Perri (2004) argue that as business cycles become more correlated, the gains from diversifying risk become smaller, and this in return decreases the holding of foreign shares from consumers, causing a deterioration in consumption smoothing. Islamaj(2008, 2009) extends this idea for different measures of risk sharing and shows supportive evidence that the increase in productivity shock correlations between a country and the rest of the world may have indeed been a reason why we fail to observe improvements in consumption smoothing as countries get more financially liberalized. The model presented here extends this idea by facilitating the study of the effects of financial liberalization on portfolio choice and consumption smoothing accounting for changes in productivity shock correlations not only between countries, but also between sectors. The relationships between liberalization and risk sharing, and productivity correlations and risk sharing are nonlinear. This nonlinearity is not merely a mathematical fact, but has important implications about consumption smoothing. This paper presents a general equilibrium framework that solves for a closed form solution and can capture the nature of these nonlinearities.

Next section explains the model. First, it explain the basic intuition for the firms' problem. In

this economy, firms choose capital, whereas consumers make portfolio choice decisions. I show the solutions for capital allocations for three different cases; first, cross-country and cross-sectoral shock correlations are zero, second, cross-sectoral shock correlations are different from zero, and third, cross-country shock correlations are positive. The results are shown in Section 3.4 and can be summarized as follows. First, more liberalization increases specialization. Second, higher cross-sectoral productivity correlations increase financial diversification and industrial specialization. Finally, higher cross-country productivity correlations, decrease portfolio diversification. For low values of impediments to trading foreign capital, firms try to increase expected profits by increasing specialization, whereas for higher values of impediments firms increase expected profits by diversifying production more. Section 3.4 also shows some implications of the model and further sensitivity analysis. Some concluding remarks and suggestions about future work are written down in the last section.

2 The Model

2.1 Domestic Firms' Problem

Consumers own k units of capital and sell it to firms with production technology:

$$\begin{aligned} y_a &= z_a A_a k_a \\ y_b &= z_b A_b k_b \end{aligned} \tag{1}$$

Consumers sell capital at price p to domestic firms and the firms decide how to allocate the capital between sector a and sector b. The sector specific z_a and z_b represent stochastic productivity shocks and A_a and A_b represent productivity coefficients. In the case of similar productivity shocks, $A_a > A_b$ will mean that in the domestic country sector a is more productive that sector b. Firms sell one share per sector and consumers buy shares at prices q_a, q_b and q_a^*, q_b^* for domestic and foreign shares respectively. Then, shock are realized and consumers consume their claims.

Firms maximize the value of their firm minus costs. The costs in this case are just the capital they purchase and the problem of the domestic firm can be written as below.

Domestic firms' problem:

$$\max_{k_a, k_b \geq 0} \{q_a + q_b - p(k_a + k_b)\} \tag{2}$$

where

$$\begin{aligned} q_a &= \sum_s \pi(s) Q(s) z_a(s) A_a k_a \\ q_b &= \sum_s \pi(s) Q(s) z_b(s) A_b k_b \end{aligned} \tag{3}$$

and $Q(s)$ is the price of a unit of output in state s, whereas $\pi(s)$ is the probability that state s will occur.

Note that an interior solution requires that $q_a = pk_a, q_b = pk_b$

In equilibrium

$$k_a + k_b = k \quad (\text{assume } k = 1)$$

First Order conditions with respect to k_a and k_b, respectively (assuming interior solution):

$$\begin{aligned} p &= \sum_s \pi(s)Q(s)z_a(s)A_a \\ p &= \sum_s \pi(s)Q(s)z_b(s)A_b \end{aligned} \tag{4}$$

which implies:

$$\sum_s \pi(s)Q(s)z_a(s)A_a = \sum_s \pi(s)Q(s)z_b(s)A_b \tag{5}$$

and if $Q(s) = u'(c(s))$ then

$$\sum_s \pi(s)u'(c(s))z_a(s)A_a = \sum_s \pi(s)u'(c(s))z_b(s)A_b \tag{6}$$

This equation will dictate the allocation of capital between sectors. Consumers will buy shares of each sector in the domestic and foreign country. The purchase of foreign shares will be subject to a tax τ that will denote impediments to purchasing foreign capital. λ_a and λ_b (λ_a^* and λ_b^*) will denote holdings of domestic (foreign) share holdings of the domestic consumer in sectors a and b, respectively. The holdings of the foreign consumer will be denoted as $\eta_a^*, \eta_b^*(\eta_a, \eta_b)$ for the holdings of the foreign (domestic) shares in sectors a and b, respectively. The share of capital that can be pinned down from equation 6 will depend on the degree of financial openness, τ. This may not be immediately obvious since consumers do not care where capital is allocated as long as they get paid p for each unit of it, and firms profit do not directly depend on the transaction costs τ. But, firms make their capital allocation decision taking as given the portfolio share choices of the consumers for each sector.

i.e., domestic firm's problem:

$$\max_{k_a, k_b \geq 0} \{q_a + q_b - p(k_a + k_b)\}$$

given, $\lambda_a, \lambda_b, \lambda_a^*, \lambda_b^*$
the same equation is still true:

$$\sum_s \pi(s)u'(c(s))z_a(s)A_a = \sum_s \pi(s)u'(c(s))z_b(s)A_b$$

Assuming the utility is exponential $u(c(s)) = -\frac{1}{\theta}e^{-\theta c(s)}$, where $c(s)$ is consumption at state s, and goods a and b are perfect substitutes, and that productivity shocks $z(s)$ come from a multivariate normal distribution, the equation above becomes:

$$(-\theta cov(c, z_a) + \mu_a)A_a = (-\theta cov(c, z_b) + \mu_b)A_b \tag{7}$$

where μ_a and μ_b are the means of the productivity shock in each sector. The standard deviations of these shocks will be assumed all equal to σ. Because the four shocks come from a multivariate normal distribution, there exist 6 correlations between them that will affect the correlations of output produced across sectors and across countries. Let's denote the correlation between productivity shock in domestic (foreign) sectors a and b as $\rho_{ab}(\rho_{a^*b^*})$, the correlation of productivity shocks across countries for sectors a and b as ρ_{aa^*} and ρ_{bb^*}, respectively, and the correlation across different sectors in different countries as ρ_{ab^*} (for sector a in the domestic country and sector b in the foreign country) and ρ_{a^*b} (for the correlation between domestic sector b and foreign sector a). For the rest of the paper we'll assume that $\rho_{ab^*} = \rho_{ab^*} = 0$, and will focus only on what happens when the productivity correlations across sectors and across countries change.

Case 1: Assuming all $\rho_{ab} = \rho_{a^*b^*} = \rho_{aa^*} = \rho_{bb^*} = \rho = 0$:

The covariances in equation 7 will be defined as:

$$\begin{aligned} cov(c, z_a) &= \lambda_a k_a A_a \sigma^2 \\ cov(c, z_b) &= \lambda_b k_b A_b \sigma^2 = \lambda_b (1 - k_a) A_b \sigma^2 \end{aligned} \tag{8}$$

and 7 can be re-written as:

$$A_a(-\theta \lambda_a k_a A \sigma^2 + \mu_a) = A_b(-\theta \lambda_b (1 - k_a) A_b \sigma^2 + \mu_b) \tag{9}$$

This equation implies that:

$$k_a = \frac{\lambda_b A_b^2}{\lambda_a A_a^2 + \lambda_b A_b^2} + \frac{\mu_a A_a - \mu_b A_b}{\theta \sigma^2 (\lambda_a A_a^2 + \lambda_b A_b^2)} \tag{10}$$

Thus, the firm chooses the capital taking portfolio holdings λ_a and λ_b as given.(analogous for k_a^*)

Note that for $A_a = A_b$ and same mean of the shocks, $k_a = \frac{1}{2}$

Case 2: assuming $\rho_{ab} = \rho_{a^*b^*} = \rho \neq 0$ and $\rho_{aa^*} = \rho_{bb^*} = 0$: i.e., cross-sectoral productivity correlations are different from zero.

In this case, the covariances in equation 7 will be defined as:

$$\begin{aligned} cov(c, z_a) &= \lambda_a k_a A_a \sigma^2 + \lambda_b A_b (1 - k_a) \rho \sigma^2 \\ cov(c, z_b) &= \lambda_a k_a A_a \rho \sigma^2 + \lambda_b (1 - k_a) A_b \sigma^2 \end{aligned} \tag{11}$$

and,

$$(-\theta cov(c, z_a) + \mu_a) A_a = (-\theta cov(c, z_b) + \mu_b) A_b \tag{12}$$

This equation implies that:

$$k_a = \frac{\lambda_b A_b (A_b - A_a \rho) + \frac{1}{\theta \sigma^2}(\mu_a A_a - \mu_b A_b)}{\lambda_a A_a^2 - (\lambda_a + \lambda_b) A_a A_b \rho + \lambda_b A_b^2} \tag{13}$$

Note that for $\rho = 0$, equation 13 turns into equation 10.

2.2 Foreign Firm's Problem

The firm in the foreign country solves an analogous problem. Denoting the value of sectors a and b in the foreign country as q_a^* and q_b^*, and the capital allocation as k_a^* and k_b^*, respectively the foreign firms' problem becomes:

$$\max_{k_a^*, k_b^* \geq 0} \{q_a^* + q_b^* - p^*(k_a^* + k_b^*)\} \tag{14}$$

which implies:

$$\sum_s \pi(s) Q^*(s) z_a^*(s) A_a^* = \sum_s \pi(s) Q^*(s) z_b^*(s) A_b^*$$

and if $Q^*(s) = u'(c^*(s))$ then

$$\sum_s \pi(s) u'(c^*(s)) z_a^*(s) A_a^* = \sum_s \pi(s) u'(c^*(s)) z_b^*(s) A_b^*$$

Making the same assumptions as for the domestic country, this equation can be re-written as:

$$(-\theta cov(c^*, z_a^*) + \mu_a^*) A_a^* = (-\theta cov(c^*, z_b^*) + \mu_b^*) A_b^* \tag{15}$$

where μ_a^* and μ_b^* are the means of the productivity shock in each sector.

Case 3: assuming $\rho_{aa^*} = \rho_{bb^*} = \rho \neq 0$ and $\rho_{ab} = \rho_{a^*b^*} = 0$: i.e., cross-country productivity correlations are different from zero.

In this case, capital allocations for each sector in the domestic and foreign countries can be pinned down by the following set of equations which are derived from the first order condition of the domestic and foreign firm's problem

$$(-\theta cov(c, z_a) + \mu_a) A_a = (-\theta cov(c, z_b) + \mu_b) A_b \tag{16}$$

$$(-\theta cov(c^*, z_a^*) + \mu_a^*) A_a^* = (-\theta cov(c^*, z_b^*) + \mu_b^*) A_b^* \tag{17}$$

where,

$$\begin{aligned} cov(c, z_a) &= \lambda_a k_a A_a \sigma^2 + \lambda_a^* A_a^* k_a^* \rho \sigma^2 \\ cov(c, z_b) &= \lambda_b (1 - k_a) A_b \sigma^2 + \lambda_b^* (1 - k_a^*) A_b^* \rho \sigma^2 \\ cov(c^*, z_a^*) &= \eta_a^* k_a^* A_a^* \sigma^2 + \eta_a A_a k_a \rho \sigma^2 \\ cov(c^*, z_b^*) &= \eta_b^* (1 - k_a^*) A_b^* \sigma^2 + \eta_b (1 - k_a) A_b \rho \sigma^2 \end{aligned} \tag{18}$$

From these equations one can find that:

$$k_a = \frac{\alpha_1 \alpha_4 - \alpha_2 \alpha_5}{\alpha_3 \alpha_4 - \alpha_5 \alpha_6} \tag{19}$$

and

$$k_a^* = \frac{\alpha_2 \alpha_3 - \alpha_1 \alpha_6}{\alpha_3 \alpha_4 - \alpha_5 \alpha_6} \tag{20}$$

where,

$$\begin{aligned}
\alpha_1 &= \frac{\mu}{\theta\sigma^2}(A_a - A_b) + \lambda_b A_b^2 + \lambda_b^* A_b A_b^* \rho \\
\alpha_2 &= \frac{\mu}{\theta\sigma^2}(A_a^* - A_b^*) + \eta_b^* A_b^{*2} + \eta_b A_b A_b^* \rho \\
\alpha_3 &= \lambda_a A_a^2 + \lambda_b A_b^2 \\
\alpha_4 &= \eta_a^* A_a^{*2} + \eta_b^* A_b^{*2} \\
\alpha_5 &= (\lambda_a^* A_a A_a^* + \lambda_b^* A_b A_b^*)\rho \\
\alpha_6 &= (\eta_a A_a A_a^* + \eta_b A_b A_b^*)\rho
\end{aligned}$$

and $\lambda_a, \lambda_b, \lambda_a^*, \lambda_b^*(\eta_a^*, \eta_b^*, \eta_a, \eta_b)$ are the holdings of the domestic and foreign portfolios of the domestic (foreign) consumer on sectors a and b, respectively.

2.3 Consumer's Problem

Consumers will choose protfolio shares for each sector that will maximize their expected utility, taking the capital allocation as given from the firm's problem. Consumer's problem becomes:

$$\max_{\lambda_a, \lambda_b, \lambda_a^*, \lambda_b^* \in [0,1]} \sum_s \pi(s) u(c(s)) \tag{21}$$

given k_a(and k_a^*)

s.t.

$$q_a \lambda_a + q_b \lambda_b + \frac{1}{(1-\tau)}(q_a^* \lambda_a^* + q_b^* \lambda_b^*) = pk \tag{22}$$

$$c(s) = \lambda_a y_a(s) + \lambda_b y_b(s) + \lambda_a^* y_a^*(s) + \lambda_b^* y_b^*(s) \tag{23}$$

Purchasing foreign capital will be subject to a tax τ. For $\tau = 0$, financial markets will be fully liberalized. As τ increases, it will be costlier for the consumer to buy shares of the foreign sectors and the consumers will bias his/her portfolio towards domestic portfolio holdings. For high enough τ, the consumers will not purchase any more shares of the foreign sectors. In this case, it can be said that financial markets are in autarky and no trade in financial assets takes place.

First order conditions for the domestic consumer (assuming all stock purchases interior):

$$\begin{aligned}
-q_a \psi + \sum_s \pi(s) u'(c(s)) \frac{\partial}{\partial \lambda_a} c(s) &= 0 \\
-q_b \psi + \sum_s \pi(s) u'(c(s)) \frac{\partial}{\partial \lambda_b} c(s) &= 0 \\
-q_a^* \psi + (1-\tau) \sum_s \pi(s) u'(c(s)) \frac{\partial}{\partial \lambda_a^*} c(s) &= 0 \\
-q_b^* \psi + (1-\tau) \sum_s \pi(s) u'(c(s)) \frac{\partial}{\partial \lambda_b^*} c(s) &= 0
\end{aligned} \tag{24}$$

and

$$c(s) = \lambda_a z_a(s) A_a k_a + \lambda_b z_b(s) A_b (1 - k_a) + \lambda_a^* z_a^*(s) A_a^* k_a^* + \lambda_b^* z_b^*(s) A_b^* (1 - k_a^*)$$

(analogous for the foreign consumer)

Three independent equations come out of these FOC's. Analogously, three more equations are derived from the foreign consumer and 4 more equations represent the stock market clearance conditions as below:

$$\begin{aligned} \lambda_a + \eta_a &= 1 \\ \lambda_b + \eta_b &= 1 \\ \lambda_a^* + \eta_a^* &= 1 \\ \lambda_b^* + \eta_b^* &= 1 \end{aligned} \tag{25}$$

From these 10 equations, one can pin down 8 portfolio choices (for the domestic and foreign consumers) and prices p and p^* that consumers sell their capital to firms for each value of impediments to foreign capital τ.

2.4 Definition of Equilibrium

Denote $s \equiv (s_a, s_b, s_a^*, s_b^*)$

An equilibrium is a set of quantities $\lambda_a(s)$, $\lambda_b(s)$, $\lambda_a^*(s)$, $\lambda_b^*(s)$, $\eta_a^*(s)$, $\eta_b^*(s)$, $\eta_a(s)$, $\eta_b(s)$, prices p, p^*, and productivity shocks $z_a(s_a)$, $z_b(s_b)$, $z_a^*(s_a^*)$, $z_b^*(s_b^*)$ which satisfy the following conditions:

1. Market clearing condition for goods:
$c_a + c_a^* = y_a + y_a^* = k_a A_a z_a + k_a^* A_a^* z_a^*$,
$c_b + c_b^* = y_b + y_b^* = k_b A_b z_b + k_b^* A_b^* z_b^*$
2. Market clearing condition for stocks:
$\lambda_a + \eta_a = 1 \qquad \lambda_a^* + \eta_a^* = 1$
$\lambda_b + \eta_b = 1 \qquad \lambda_b^* + \eta_b^* = 1$

3 Symetric case

Consider the case when both countries are symetric:

$$\begin{aligned} A_a &= A_b^* = A \\ A_b &= A_a^* = B \\ p &= p^* \end{aligned} \tag{26}$$

which implies:

$$\begin{aligned} k_a &= 1 - k_b = k_b^* = 1 - k_a^* \\ \lambda_a &= 1 - \eta_a = \eta_b^* = 1 - \lambda_b^* \\ \lambda_b &= 1 - \eta_b = \eta_a^* = 1 - \lambda_a^* \end{aligned} \tag{27}$$

In this case, the equilibrium λ_a and λ_b can be pinned down by the first two independent equations coming out of the FOC's. The next section will show the portfolio holdings, capital allocations and consumption smoothing for each value of τ. More sensitivity analysis will show what happens when cross-country and cross-sectoral productivity correlations change.

4 Results

4.1 Implications of the Model

This section discusses some theoretical implications of this model. First, it studies the holdings of shares for each sector and capital allocation across sectors as the country gets more financially liberalized. As liberalization increases, the consumer holds less shares of the domestic sectors and starts buying more shares of the foreign sectors. Also, with liberalization more capital is allocated to the most productive sector. Next, it discusses the implications of this model about the relationship between financial liberalization and consumption smoothing. Then, it looks at what happens to portfolio shares for each sector as the productivity correlations change. Given the complexity of this model, it is easy to distinguish between the effects of a change in cross-country and cross-sector productivity correlations. It can be shown that the more similar the productivity processes between countries, the lower the amount of asset trade among countries. The intuition would be that gains from sharing risks would be smaller. On the other hand, the higher the correlation between productivity processes between sectors in each country, the lower the portfolio shares of the domestic sectors and the higher the incentives of individuals to share income risks abroad. This section aslo discusses the effects of a change in cross-contry and cross-sector productivity correlations on capital allocation across sectors.

4.1.1 Financial Liberalization, Portfolio Holdings and Industrial Specialization

This model provides a framework to relate financial liberalization, portfolio shares and specialization. Most importantly, in this framework countries do not necessarily fully specialize in one good, even if they can produce it more efficiently. Incomplete specialization is achieved because of uncertainty in technology. As explained above, individuals in each country face uncertainty in both sectors. If they specialize in only one sector, then there exists a possibility that they will be faced with a bad shock and not be able to meet their consumption needs. If there was financial autarky, countries would share this risk internally by allocating capital in both sectors. When financial markets are open, the consumer engages in trading claims to production with the other country. In this case, the consumer are provided with an extra insurance and this "insured wealth" allows the countries to expand production in their most productive sector without jeopardizing their consumption. But, financial markets serve only to *share* risks efficiently, not to *eliminate* them (even when markets are complete). Thus, production risk is still present, and as a result we do not see full specialization in production.

Figure 3.1 shows what happens to portfolio holdings as impediments to trade in foreign capital decrease. The parameters of the model are similar to those in Chapter 2; $\mu = 2, \sigma^2 = 0.01, \theta = 1$ and $A_a = 1.001, A_b = 1$ (See Chapter 2 for more details on how to interpret these values). Note that $A_a > A_b$, meaning that sector a is more productive than sector b. Moving left on the horizontal axis represents less frictions in buying foreign assets and can be represented as financial liberalization. As countries become more open, the consumer holds less shares of the domestic sectors. Purchases of foreign sectors also increase as financial openness increases (not shown in the figure). Full liberalization corresponds to a portfolio share of 1/2 for each sector as in Heathcote and Perri (2004). The results are similar for holdings in sector b and foreign holdings are 0 for restrictive impediments, whereas go up to 1/2 for fully liberalized markets.

Figure 3.2 shows what happens to the capital share in sector a as financial restrictions decrease. As discussed, for high values of τ, which correspond to closed financial markets, individuals in each country engage in the production of both goods in order to avoid the risks stemming from volatility in each sector. As impediments to trade in foreign capital decrease and consumers start buying shares of the foreign sectors (as shown in Figure 3.1), consumers in each country enjoy some extra insurance. Firms, which care about their shareholders, respond by expanding production in the most efficient sector. The figure shows what happens to capital allocation in sector "a" in the domestic country (capital allocation in sector "b" is just one minus capital in sector "a"). As expected, although there is some specialization, the countries do not fully specialize because there still exists some production risk that is not fully ensured. The results are analogous for the foreign country (note that in the foreign country sector b is more efficient).

This framework allows us to track changes in portfolio choice, capital supply and other variables as countries liberalize their accounts. Everything else equal, there exists a non-increasing relationship between liberalization and portfolio shares, as well as a monotonic relationship between openness and capital allocation in each sector. Note that this relationship is nonlinear and the more liberalized the country the larger the effect of financial liberalization on capital allocation. Later the model will study the effects of changes in cross-country and cross-sector productivity correlations on portfolio shares and capital allocation.

4.1.2 Financial Liberalization, Productivity Shock Correlation and Consumption Risk Sharing

The literature on consumption risk sharing falls into three broad categories. The first strand emphasizes that in complete financial markets, marginal utility growth should be equated across countries so that consumption growth rates should be highly correlated. This correlation-based approach predicts that, absent financial market frictions, not only should consumption be highly correlated across countries, but it also should be more correlated than output (Backus, Kehoe and Kydland (1995)). A second strand of the empirical consumption risk sharing literature has focused on regression-based measures. These studies emphasize an alternative prediction of the complete markets model: fluctuation in relative marginal utility growth should be independent of idiosyncratic risk (as measured by relative output growth rates). Therefore, the coefficient of a regression of relative consumption growth on relative output growth should be low under high degrees of financial liberalization (Mace (1991), Cochrane (1991), Lewis (1996)). The acknowledgement that real financial markets are likely to be incomplete has led researchers to adopt a more pragmatic approach in applied work, and argue that the coefficient of this regression can be of interest in itself and that it should be interpreted as a measure of the deviation from the complete markets outcome (Asdrubali, Sorensen and Yosha (1996), Crucini (1999). Another prediction in the theoretical literature of consumption risk sharing is that volatility of consumption should be low in financially open settings (some recent studies have been looking at the ratio of consumption volatility and output volatility to better control for output crises) (Obstfeld and Rogoff (1996), Kose, Prasad and Terrones (2003).

Figure 3.3 shows what happens to the measures of consumption smoothing as restrictions to trade in capital shares decrease (Consumption volatility also decreases as the country becomes more liberalized (not shown in the Figure)). The dotted thick blue line shows $corr(c, c^*)$. Standard the-

ories predict that in financially open environments consumption should be highly correlated across countries. The solid red line shows $corr(c, y)$, the correlation between domestic consumption and domestic output. A lower correlation would correspond to better consumption risk sharing. For all these measures of consumption smoothing, more liberalization means better smoothing, everything else equal. It should be noted that this relationship is highly nonlinear and the actual level of impediments to capital matters. For example, for high levels of τ in Figure 3.3 as impediments to trade in foreign capital decline, there is little change in $corr(c, y)$ (dotted line). Only for low τ do we observe an increase in consumption smoothing as restrictions to trading capital shares decrease.

All these available measures of consumption risk sharing are associated with potential advantages and disadvantages (see Artis and Hoffmann (2004) for a more complete discussion). So far, the literature has not considered any one measure better than the others, but it has been using them interchangeably. This framework can highlight some of the differences across these measures. First, as openness increases consumption smoothing improves with an increasing rate for the $corr(c, y)$ measure (lower $corr(c, y)$ means better consumption smoothing), whereas improves at a decreasing rate when $corr(c, c^*)$ is used as a measure of conumption risk sharing. Second, whereas $corr(c, c^*)$ can increase from 0 to 1 as openness increases, $corr(c, y)$ declines only from 1 to approximately 0.7 ($\rho = 0$). Thus, although both measures suggest better consumption smoothing as liberalization increases, the effects of an incremental change in liberalization differ both qualitatively and quantitatively across measures.

4.2 The Role of Productivity Shock Correlations and Further Sensitivity Analysis

What happens if productivity shock correlations between countries and/or sectors are greater than zero? Figure 3.4 shows a scenario where productivity correlations across sectors, for each country, are different from zero, but cross country productivity shock correlations are zero, i.e., $\rho_{ab} = \rho_{a^*b^*} = \rho = 0.2$, but $\rho_{aa^*} = \rho_{bb^*} = 0$[1]. Again, the solid line shows the case when all productivity correlations are zero and the dotted line shows the case when productivity correlations across sectors for each country are positive. As can be seen from the figure, the more similar the productivity processes across sectors the lower the shares of the domestic sector a held (the curve shifts down). The intuition would be because the gains to diversifying risk across sectors are lower, so the consumers choose to diversify risks across countries.

Figure 3.5 shows what happens to capital allocation for a low and high correlation of productivities between sectors within a country as impediments to capital decrease. In this case sector a is more productive in the home country and sector b is more productive in the foreign country. Again, we see some specialization as the countries liberalize their capital accounts, but not full specialization. For each τ, the green dashed line (which corresponds to higher shock correlations) shows a higher level of specialization than the blue solid line (low level of productivity correlations). The intuition would be that the more similar the productivity processes, the higher the probability of experiencing the same loss however you diversify, so the individuals have a higher incentive to focus their production in the sector that will give them more output. Again, not all risks are eliminated,

[1] Islamaj (2009) calculates the average productivity correlations with the rest of the world to be approximately 0.07 for developing countries and around 0.2 for developed ones.

so full specialization will not occur.

Figure 3.6 shows what happens for every level of τ when all productivity correlations are zero (solid line) and when cross-country productivity correlations for each sector are positive, i.e., $\rho_{aa^*} = \rho_{bb^*} = \rho = 0.2$. As expected, the consumer holds a higher share of the domestic sectors in this case because gains from risk sharing are smaller. Higher cross-country productivity correlations provide fewer incentives to buy shares of the foreign country.

Figure 3.7 maps measures of consumption smoothing on impediments to trade in foreign capital. It also shows what happens to these relationships as cross-country productivity correlations increase (ρ_{aa^*} and ρ_{bb^*}). As expected, higher productivity correlations deteriorates consumption smoothing. $Corr(c, y)$ jumps up and this will correspond to a decline in consumption risk sharing, whereas $corr(c, c^*)$ shifts down and this will mean a deteriaration in consumption smoothing as well. For higher cross-sector productivity correlation, the story is exactly the opposite and consumption risk sharing improves as countries purchase more foreign assets.

Figure 3.8 shows what happens for every level of τ when all productivity correlations are zero (solid line) and when both cross-country and cross-sectoral productivity correlations are positive, i.e., $\rho_{aa^*} = \rho_{bb^*} = \rho_{ab} = \rho_{a^*b^*} = \rho = 0.2$. In all cases the curves jump upwards. But we read these measures differently. We say that a country is better able to smooth consumption if it experiences a higher $corr(cc^*)$ and/or a lower $corr(c, y)$. In Figure 3.8 we can see that an increase in productivity correlation with the rest of the world deteriorates consumption risk sharing if the measure of consumption smoothing is $corr(c, y)$, but it improves consumption risk sharing if one uses $corr(cc^*)$. Thus, the effect of a change in cross-sector and/or cross-country productivity correlation is different across measures of consumption smoothing. As discussed, so far the literature has treated all these measures as equivalent, but as seen in Figure 3.8 they can respond differently to different changes in productivity shock correlations. More research on differences across measures of consumption risk sharing is necessary.

Figure 3.9 shows what happens to capital allocations as cross-country productivity correlations increase. In this case, there are two opposing effects at work. First, as countries become more similar, there are fewer incentives to diversify abroad and this will lead to more diversification of production at home. On the other hand, as cross-country correlations increase, aggregate volatility increases, which in turn will make portfolio diversification less profitable. Firms at home will prefer to switch capital to the more productive sector a as the consumer buys less shares of sector a abroad. Figure 3.9 shows that for high enough impediments to purchasing foreign assets, the former effect dominates and firms diversify production at home. For low restrictions, as cross-country productivity shock correlations increase, firms put more capital to the more productive sector a.

Figure 3.10 shows what happens to output correlations as the country liberalizes its financial markets. Because more liberalization provides extra consumption insurance and allows countries to specialize in their most productive sector, output across countries will become less correlated as countries open their financial markets. This is in line with the empirical findings of Kalemli-Ozcan, Papaioannou and Peydro (2009). Figure 3.11 shows that output volatility also increases as impediments to purchasing foreign capital decrease, which is the result of Obstfeld (1994) and Evans and Hnatkovska (2007). For the parameter values used in this paper, Output volatility has increased by more than 3%.

This study develops a framework that allows us to map financial liberalization and portfolio choice in a two dimensional framework. It shows a link between financial openness and output volatility. As countries become more financially open and engage in asset trade, they will use the extra consumption insurance provided by the international asset markets to take more risks and will expand production in their most productive sector. In return output volatility will increase and cross-country output correlations will decline. This model also allows us to map changes in specialization and consumption risk sharing as financial markets become more open. *Ceteris parabus*, there exists a decreasing relationship between liberalization and portfolio shares in each sector, as well as between openness and consumption risk sharing. This is what standard theories suggest, but there are a few caveats to these results. First, higher cross-country (cross-sector) productivity correlations imply higher (lower) portfolio shares in the domestic sectors. Second, if a country has liberalized, but at the same time its productivity correlations with the rest of the world has increased, we may see an increase in portfolio shares, as opposed to a decrease as standard theories suggest.

Another caveat goes to the measures of consumption smoothing. I have picked two measures of consumption risk sharing interchangeably used in the literature. The model shows that these measures respond differently to changes in productivity shock correlation between countries and between sectors. This calls for a more careful consideration of the measures of consumption risk sharing in the literature.

Further sensitivity analysis tries several things. First, a higher θ, the risk sharing coefficient corresponds to higher share holdings of the domestic firms and less specialization for each τ (except for $\tau = 0$ where we see full diversification of assets). A higher mean of the productivity shock z in sector a in domestic country, corresponds to more specialization in home country than abroad and more shares of the domestic firms. A higher mean in both sectors in home country corresponds to more shares in both domestic sectors.

This paper presents a simple model of international asset trade that shows that more financial liberalization will provide countries with more consumption insurance. This extra insurance, in return, will enable countries to take extra risks and expand production in their most productive sector. Output volatility will increase and output across countries will be less correlated. These results are supported by recent empirical work.

5 Conclusions and Future Work

International financial markets have become increasingly integrated in the past two decades. Recent empirical studies (Kalemli-Ozcan, Sorensen and Yosha (2003) and Kalemli-Ozcan, Papaioannou and Peydro (2009)) show that a higher degree of financial integration is associated with more specialization and lower output correlations across countries. This paper presents a simple model of international financial trade that relates financial openness with industrial specialization and output volatility. As countries open their financial markets and engage in asset trade with foreign countries, they increase their consumption insurance. This extra insurance would allow the countries to take more risks, and, as a result, they will expand the production on their most efficient sectors. In return, they will experience a higher output volatility. Also, lower cross-country output correlations will be observed.

This framework has important implications for the consumption smoothing literature. Theoretical studies predict that country portfolios should be heavily biased towards foreign assets, leading to highly correlated consumption growth rates between countries and low correlations between growth of domestic consumption and domestic output. However, empirical studies investigating the effects of financial integration on consumption smoothing are at best inconclusive, failing to find support to the theoretical predictions of this literature. This paper develops a two-country, two-sector framework with uncertainty in production to reconcile theory and data. In particular, this paper analytically solves for the equilibrium consumption based measures of risk sharing . The two countries receive stochastic productivity shocks which are then combined with capital to produce final consumption goods. Consumers own domestic capital and can buy shares of foreign capital subject to a tax which represents market restrictions in this model.

In this setup, consistent with the data, consumption based measures of risk sharing may deteriorate even when the countries liberalize their financial markets substantially. The model suggests that financial market frictions, and increases in productivity shock correlations between two countries, may help explain why we fail to see improvements in consumption risk sharing as countries have become more financially integrated. There exists a monotonic relationship between financial liberalization and consumption smoothing, i.e., more liberalization means that countries are able to better smooth their consumption risks, but this relationship is nonlinear and the actual level of impediments to buying foreign capital matters for the extent of risk sharing. Further, an increase in productivity shock correlations between two countries as financial markets liberalize may reduce the gains of diversifying consumption and cause the measures of consumption risk sharing to deteriorate. The paper shows how the relationship between financial integration and consumption smoothing is qualitatively consistent with facts documented in the empirical literature in this area.

This paper also highlights some differences across different measures of consumption smoothing. So far, the literature has not differentiated between these measures and has been using them intechangeably. This study suggest that there are both qualitative and quantitative differences across the relationships between financial liberalization and these measures and consumption smoothing. For example, focusing on cross-country consumption correlations consumption smoothing would improve at a decreasing rate as liberalization increases and it can improve from 0 (low consumption smoothing) to 1 (high consumption smoothing). On the other hand, using correlations between domestic consumption and output as the measure of consumption risk sharing, consumption smoothing will improve with an increasing rate as the country becomes more open. Also, consumption smoothing will only improve from 1 (low consumption smoothing) to approximately 0.7 (low consumption smoothing). Furthermore, cross-country and cross-sector productivity correlations affect these measures differently. An increase in both cross-country and cross-sector productivity correlations will deteriorate consumption smoothing according to the response of the correlation between domestic consumption and domestic output, but will improve consumption smoothing according to the response of the cross-country consumption correlations. Further research studying the differences across these measures can be done in the future.

Also, future work should consist in analyzing further the effects of productivity shock correlations between countries and between sectors on consumption smoothing. It is interesting and more realistic to investigate these relationships adding investment to the analysis. Also, dynamic considerations, abstracted in this study are important directions in which this literature should be further developed. In the end, assuming that the goods are not perfect substitutes and solving this model can help one analyze the relationship between liberalization and trade and test this theory

empirically.

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6 Appendix

6.1 Solving the autarky case and showing an application of Stein's Lemma

$\max_{n_a,} E[u(c)]$

st. $c = k_a A_a z_a + k_b A_b z_b$ and $k_a + k_b = 1$

FOC: $E[u'(c)(A_a z_a - A_b z_b)] = 0$
$A_a E[u'(c) z_a] = A_b E[u'(c) z_b]$
$A_a[cov(u'(c), z_a) + E[u'(c)]E[z_a]] = A_b[cov(u'(c), z_b) + E[u'(c)]E[z_b]]$

applying Stein's Lemma:

$A_a[-\theta cov(c, z_a) + E[z_a]] = A_b[-\theta cov(c, z_b) + E[z_b]]$
$cov(c, z_a) = A_a k_a \sigma^2 + (1 - k_a) A_b \rho \sigma^2$
$cov(c, z_b) = k_a A_a \rho \sigma^2 + (1 - k_a) A_b \sigma^2$

$A_a[-\theta(A_a k_a \sigma^2 + (1 - k_a) A_b \rho \sigma^2) + \mu_a] = A_b[-\theta(k_a A_a \rho \sigma^2 + (1 - k_a) A_b \sigma^2) + \mu_b]$

solving: $k_a = \frac{A_b(\mu_b - \theta\sigma^2 A_b) - A_a(\mu_a - \theta\sigma^2 \rho A_b)}{-\theta A_a(\sigma^2 A_a - \sigma^2 \rho A_b) + \theta A_b(-\sigma^2 A_b + \sigma^2 \rho A_a)}$

6.2 Definitions

$c = \lambda_a k_a A_a z_a + \lambda_a^* k_a^* A_a^* z_a^* + \lambda_b k_b A_b z_b + \lambda_b^* k_b^* A_b^* z_b^*$
$y = k_a A_a z_a + k_b A_b z_b$

$corr(c, y) = \frac{cov(c,y)}{\sigma_c \sigma_y}$

$cov(c, y) = \lambda_a k_a^2 A_a^2 \sigma_a^2 + \lambda_a k_a k_b A_a A_b \rho_{ab} \sigma_a \sigma_b +$
$\lambda_a^* k_a^* k_a A_a^* A_a \rho_{aa^*} \sigma_{a^*} \sigma_a + \lambda_a^* k_a^* k_b A_a^* A_b \rho_{a^* b} \sigma_{a^*} \sigma_b +$
$\lambda_b k_a k_b A_a A_b \rho_{ab} \sigma_a \sigma_b + \lambda_b k_b^2 A_b^2 \sigma_b^2 +$
$\lambda_b^* k_b^* k_a A_b^* A_a \rho_{ab^*} \sigma_{b^*} \sigma_a + \lambda_b^* k_b^* k_b A_b^* A_b \rho_{bb^*} \sigma_{b^*} \sigma_b$

$$\sigma_y^2 = k_a^2 A_a^2 \sigma_a^2 + 2k_a k_b A_a A_b \rho_{ab} \sigma_a \sigma_b + k_b^2 A_b^2 \sigma_b^2$$

$$\begin{aligned}\sigma_c^2 = {} & \lambda_a^2 k_a^2 A_a^2 \sigma_a^2 + 2\lambda_a \lambda_a^* k_a^* k_a A_a^* A_a \rho_{aa^*} \sigma_{a^*} \sigma_a + \\ & 2\lambda_a \lambda_b k_a k_b A_a A_b \rho_{ab} \sigma_a \sigma_b + 2\lambda_a \lambda_b^* k_b^* k_a A_b^* A_a \rho_{ab^*} \sigma_{b^*} \sigma_a + \\ & (\lambda_a^*)^2 k_a^{*2} A_a^{*2} \sigma_{a^*}^2 + 2\lambda_a^f \lambda_b k_a^* k_b A_a^* A_b \rho_{a^* b} \sigma_{a^*} \sigma_b + \\ & 2\lambda_a^* \lambda_b^* k_a^* k_b^* A_a^* A_b^* \rho_{a^* b^*} \sigma_{a^*} \sigma_{b^*} + \lambda_b^2 k_b^2 A_b^2 \sigma_b^2 + \\ & 2\lambda_b \lambda_b^* k_b^* k_b A_b^* A_b \rho_{bb^*} \sigma_{b^*} \sigma_b + (\lambda_b^*)^2 k_b^{*2} A_b^{*2} \sigma_{b^*}^2\end{aligned}$$

$$c = \lambda_a k_a A_a z_a + \lambda_a^* k_a^* A_a^* z_a^* + \lambda_b k_b A_b z_b + \lambda_b^* k_b^* A_b^* z_b^*$$

$$c^* = \eta_a^* k_a^* A_a^* z_a^* + \eta_a k_a A_a z_a + \eta_b^* k_b^* A_b^* z_b^* + \eta_b k_b A_b z_b$$

$$corr(c, c^*) = \frac{cov(c, c*)}{\sigma_c \sigma_{c^*}}$$

$$cov(c, c^*) = E[cc^*] - E[c]E[c^*]$$

$$\begin{aligned}cov(c, c^*) = {} & \lambda_a \eta_a^* k_a k_a^* A_a A_a^* \rho_{aa^*} \sigma_a \sigma_a^* + \lambda_a \eta_a k_a^2 A_a^2 \sigma_a^2 + \\ & \lambda_a \eta_b^* k_a k_b^* A_a A_b^* \rho_{ab^*} \sigma_a \sigma_b^* + \lambda_a \eta_b k_a k_b A_a A_b \rho_{ab} \sigma_a \sigma_b + \\ & \eta_a^* \lambda_b k_a^* k_b A_a^* A_b \rho_{a^* b} \sigma_a^* \sigma_b + \eta_a \lambda_b k_a k_b A_a A_b \rho_{ab} \sigma_a \sigma_b + \\ & \lambda_b \eta_b^* k_b k_b^* A_b A_b^* \rho_{bb^*} \sigma_b \sigma_b^* + \lambda_b \eta_b k_b^2 A_b^2 \sigma_b^2 + \\ & \lambda_a^* \eta_a^* k_a^{*2} A_a^{*2} \sigma_a^{*2} + \lambda_a^* \eta_a k_a k_a^* A_a A_a^* \rho_{aa^*} \sigma_a \sigma_a^* + \\ & \lambda_a^* \eta_b^* k_a^* k_b^* A_a^* A_b^* \rho_{a^* b^*} \sigma_a^* \sigma_b^* + \lambda_a^* \eta_b k_a^* k_b A_a^* A_b \rho_{a^* b} \sigma_a^* \sigma_b + \\ & \eta_a^* \lambda_b^* k_a^* k_b^* A_a^* A_b^* \rho_{a^* b^*} \sigma_a^* \sigma_b^* + \eta_a \lambda_b^* k_a k_b^* A_a A_b^* \rho_{ab^*} \sigma_a \sigma_b^* + \\ & \eta_b^* \lambda_b^* k_b^{*2} A_b^{*2} \sigma_b^{*2} + \eta_b \lambda_b^* k_b k_b^* A_b A_b^* \rho_{bb^*} \sigma_b \sigma_b^*\end{aligned}$$

$$\begin{aligned}\sigma_{c^*}^2 = {} & \eta_a^{*2} k_a^{*2} A_a^{*2} \sigma_a^{*2} + 2\eta_a^* \eta_a k_a k_a^* A_a A_a^* \rho_{aa^*} \sigma_a \sigma_a^* + \\ & 2\eta_a^* \eta_b^* k_a^* k_b^* A_a^* A_b^* \rho_{a^* b^*} \sigma_a^* \sigma_b^* + 2\eta_a^* \eta_b k_a^* k_b A_a^* A_b \rho_{a^* b} \sigma_a^* \sigma_b + \\ & \eta_a^2 k_a^2 A_a^2 \sigma_a^2 + 2\eta_a \eta_b^* k_a k_b^* A_a A_b^* \rho_{ab^*} \sigma_a \sigma_b^* + \\ & 2\eta_a \eta_b k_a k_b A_a A_b \rho_{ab} \sigma_a \sigma_b + \eta_b^{*2} k_b^{*2} A_b^{*2} \sigma_b^{*2} + \\ & 2\eta_b^* \eta_b k_b k_b^* A_b A_b^* \rho_{bb^*} \sigma_b \sigma_b^* + \eta_b^2 k_b^2 A_b^2 \sigma_b^2\end{aligned}$$

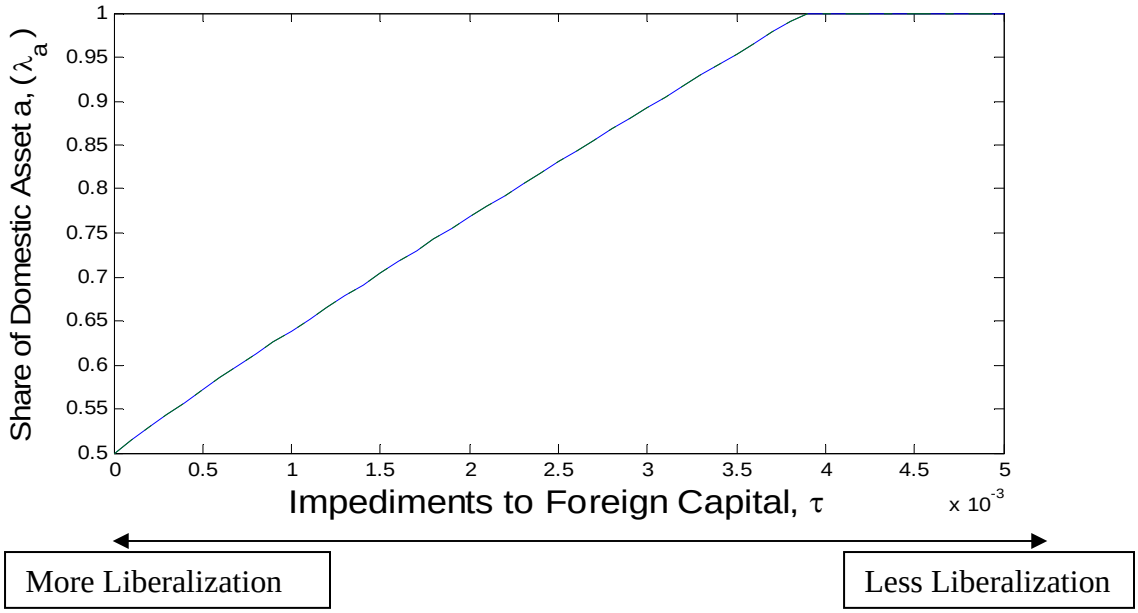
$$y = k_a A_a z_a + k_b A_b z_b$$
$$y^* = k_a^* A_a^* z_a^* + k_b^* A_b^* z_b^*$$

$$corr(y, y^*) = \frac{cov(y, y^*)}{\sigma_y \sigma_{y^*}}$$

$$cov(y, y^*) = k_a k_a^* A_a A_a^* \rho_{aa^*} \sigma_a \sigma_a^* + k_a k_b^* A_a A_b^* \rho_{ab^*} \sigma_a \sigma_b^* + k_b k_a^* A_b A_a^* \rho_{a^* b} \sigma_b \sigma_a^* + k_b k_b^* A_b A_b^* \rho_{bb^*} \sigma_b \sigma_b^*$$

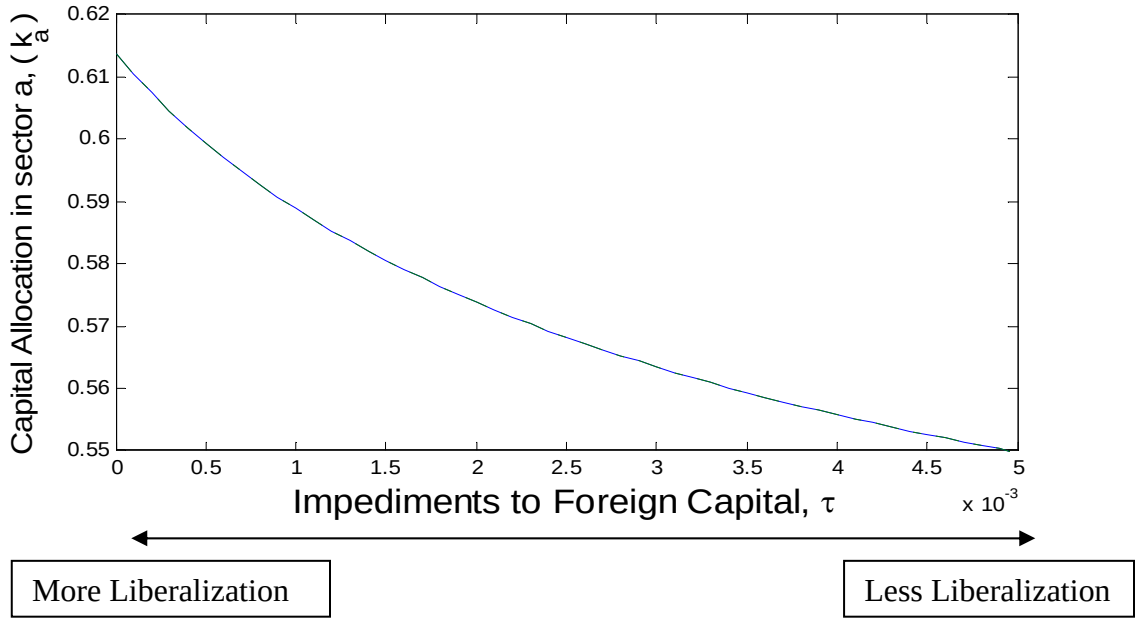
$$\sigma_{y^*}^2 = k_a^{*2} A_a^{*2} \sigma_a^{*2} + 2k_a^* k_b^* A_a^* A_b^* \rho_{a^* b^*} \sigma_{a^*} \sigma_{b^*} + k_b^{*2} A_b^{*2} \sigma_b^{*2}$$

Figure 3.1: Liberalization and Home Portfolio Share



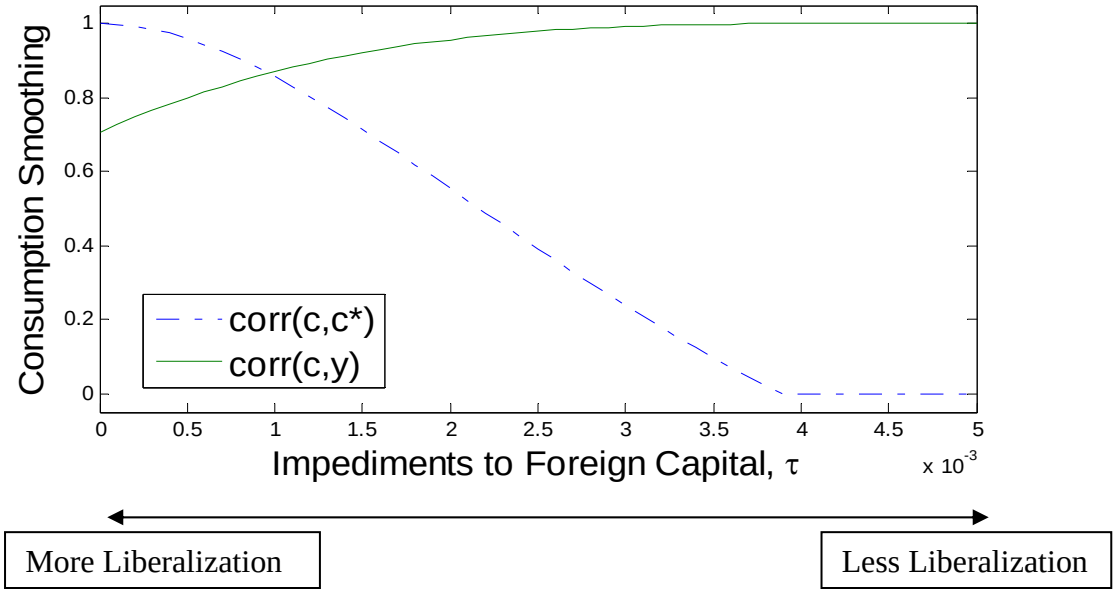
Note: The x-axis represents impediments to trade in foreign capital. On the right side, frictions in capital markets are high and the markets can be closed (no trade in financial assets takes place). On the right side, frictions in capital markets are high and the markets cans be closed (no trade in financial assets takes place). As taxes to foreign capital approach to zero, we can see the share of domestic assets in home portfolio decrease.

Figure 3.2: Liberalization and Specialization



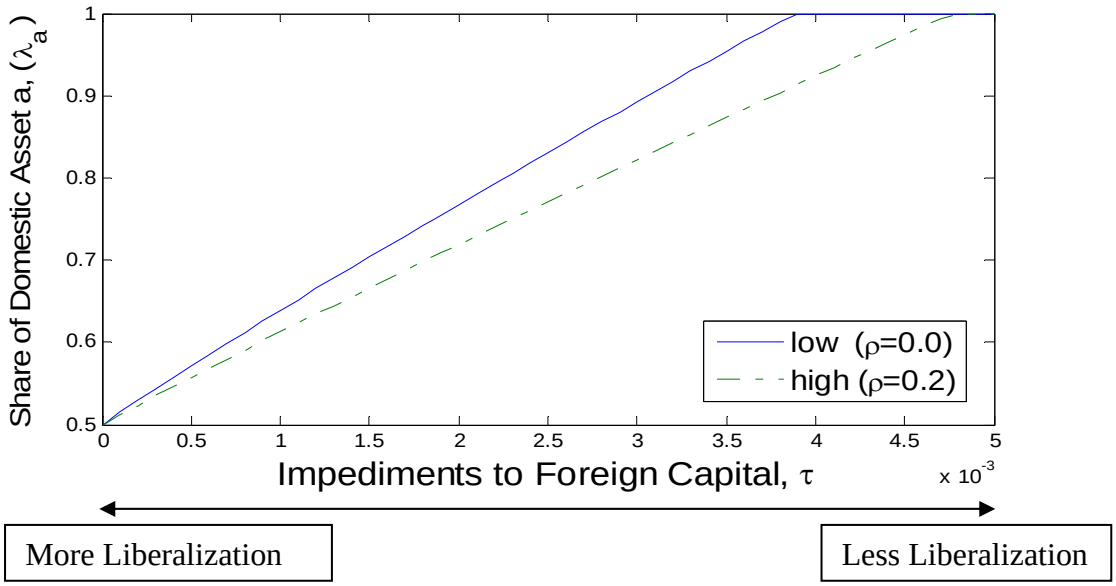
Note: The x-axis represents impediments to trade in foreign capital. As the country gets more financially liberalized more capital gets allocated in the most productive sector (y-axis).

Figure 3.3: Liberalization and Consumption Smoothing



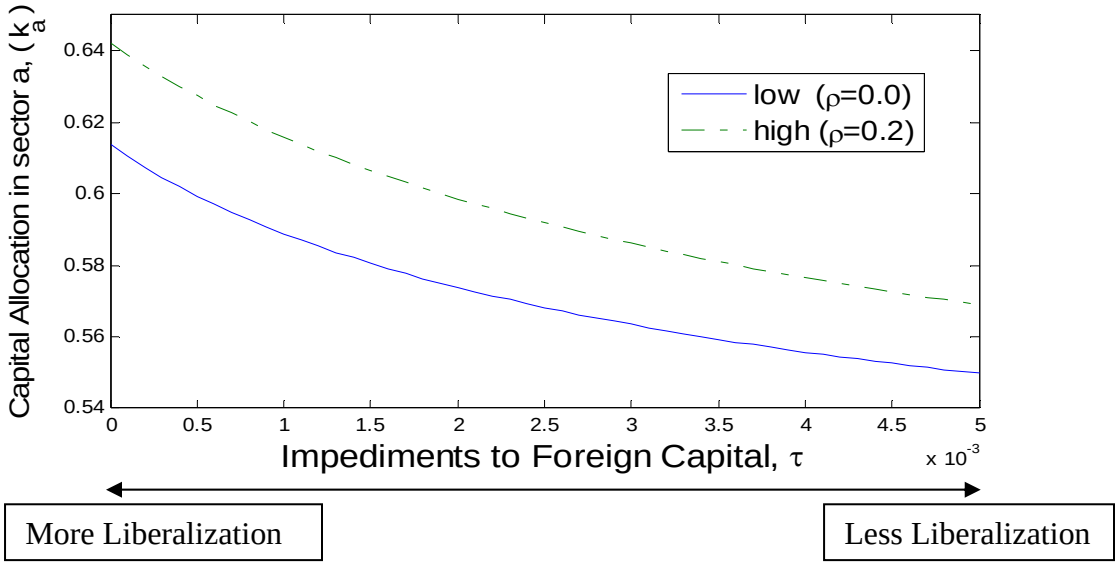
Note: The x-axis represents impediments to trade in foreign capital. The y-axis shows consumption smoothing. As consumptions correlation, corr(c,c*), increases consumption smoothing improves, whereas, as correlation between domestic consumption and output, corr(c,y), increases consumption smoothing deteriorates. For high impediments, financial markets are closed and there is no change in consumption smoothing as countries liberalize.

Figure 3.4: Portfolio Holdings and Cross-Sector Productivity Correlations



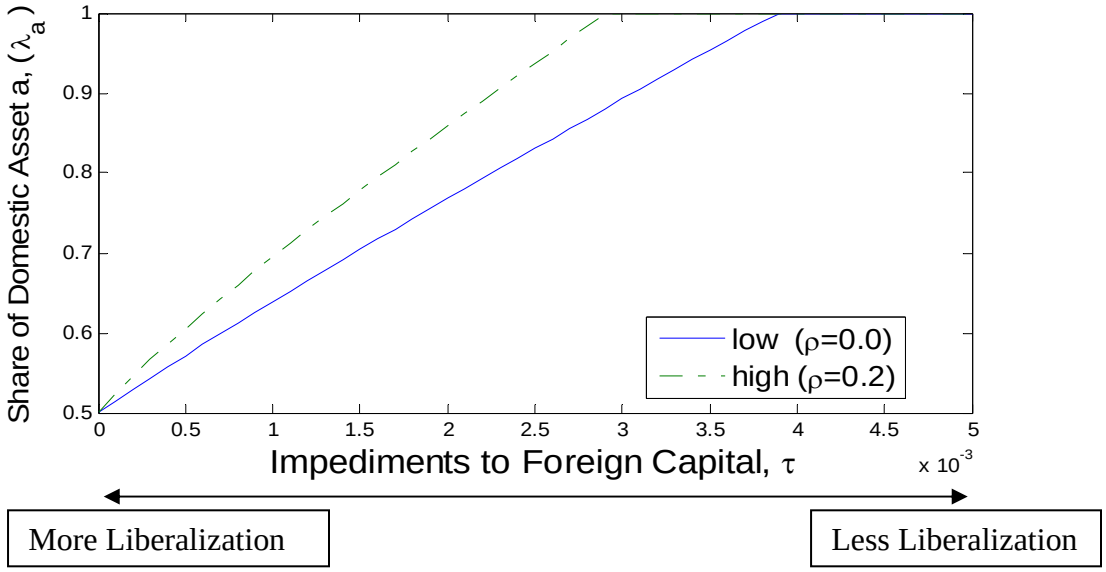
Note: The x-axis represents impediments to trade in foreign capital. For higher cross-sector productivity correlations consumers hold less shares of domestic asset in home portfolio.

Figure 3.5: Specialization and Cross-Sector Productivity Correlations



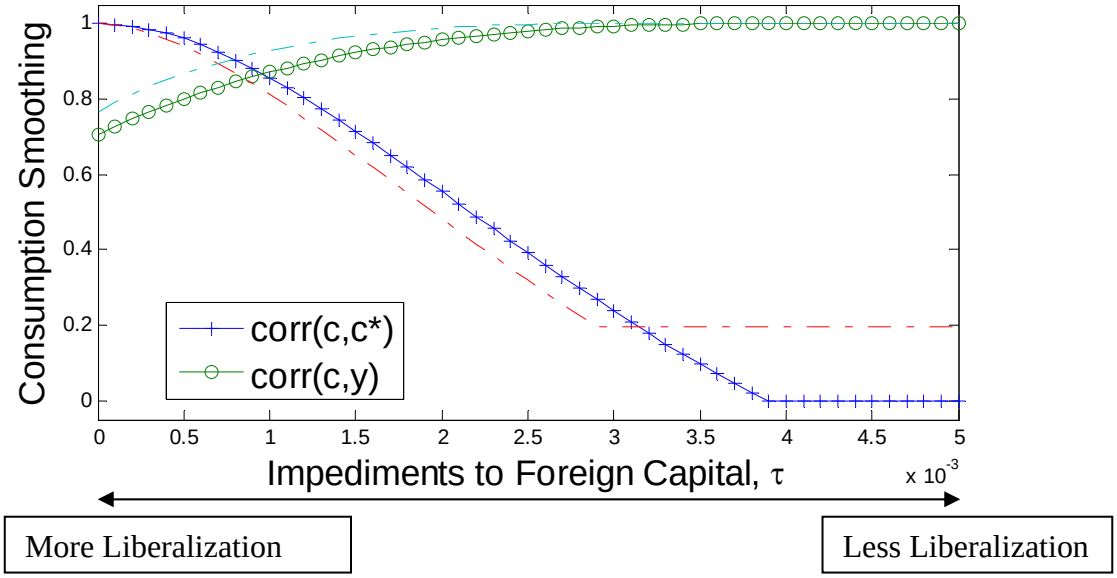
Note: The x-axis represents impediments to trade in foreign capital. Y-axes shows that as productivity processes across sectors become more similar, countries allocate more capital to the most productive sector.

Figure 3.6: Portfolio Holdings and Cross-Country Productivity Correlations



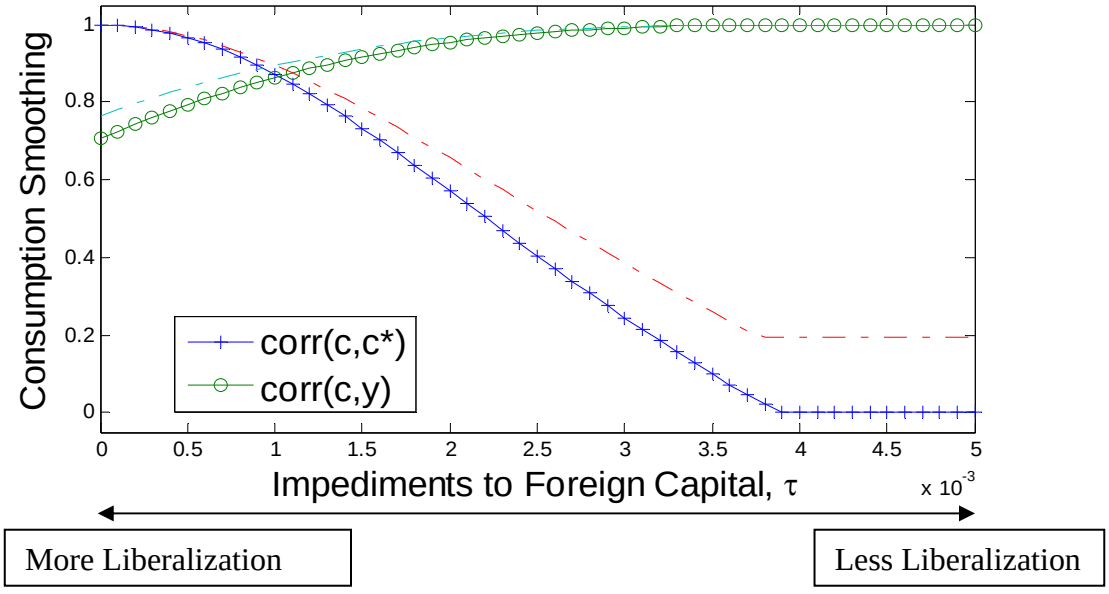
Note: The x-axis represents impediments to trade in foreign capital. On the right side, frictions in capital markets are high and the markets can be closed (no trade in financial assets takes place). On the right side, frictions in capital markets are high and the markets cans be closed (no trade in financial assets takes place). As taxes to foreign capital approach to zero, we can see share holdings of domestic asset in home portfolio decrease. For higher cross-country productivity correlations, consumers hold less foreign assets.

Figure 3.7: Consumption Smoothing and Cross-Country Productivity Correlations



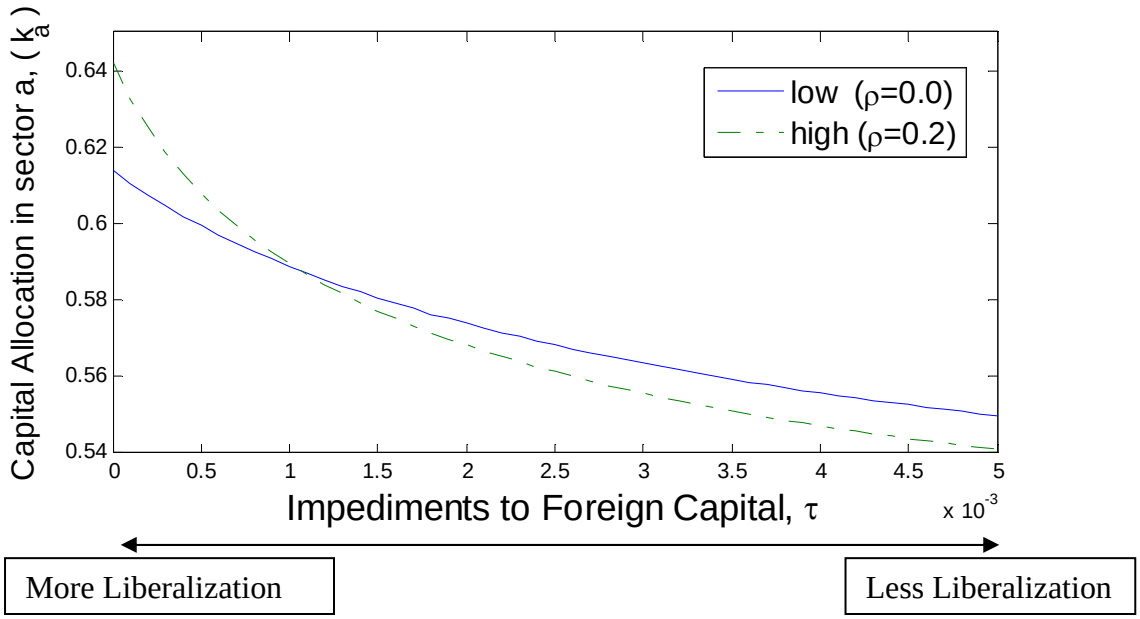
Note: The x-axis represents impediments to trade in foreign capital. For higher cross-country productivity correlations both measures of consumption risk sharing show deterioration in consumption smoothing (corr(c,c*) shifts down and corr(c,y) shifts up).

Figure 8: Productivity Correlations – Horse Race



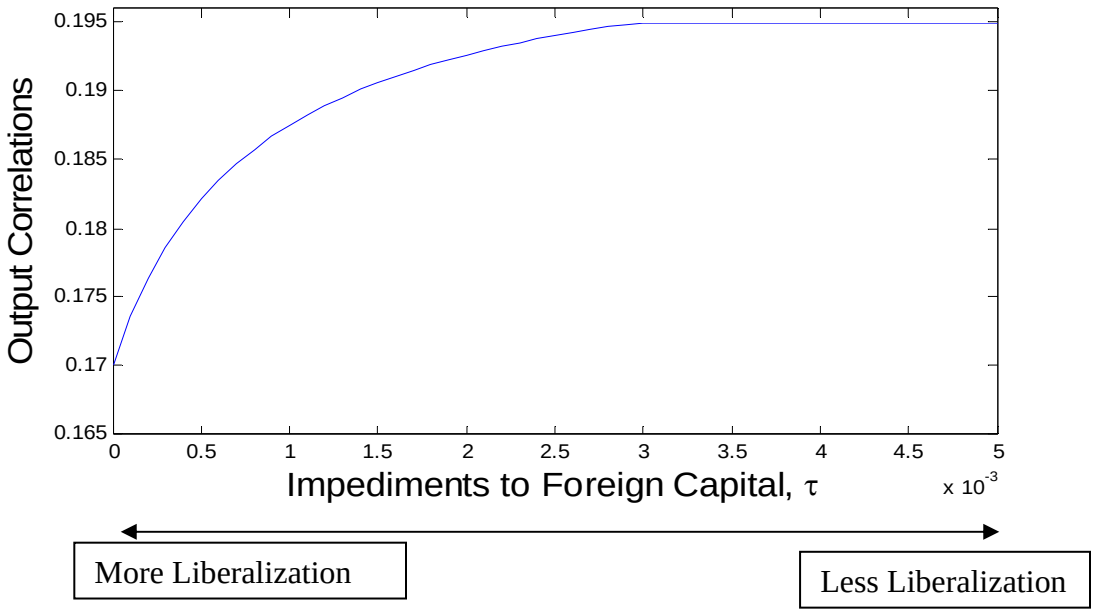
Note: The x-axis represents impediments to trade in foreign capital. Y-axis shows what happens to both measures of consumption smoothing as both cross-country and cross-sector productivity correlations increase. Both curves jump up, but consumption smoothing has deteriorated according to the corr(c,y) measure (dotted turquoise line) and improved according to the corr(c,c*) measure (dotted red line).

Figure 3.9: Specialization and Cross-Country Productivity Correlations



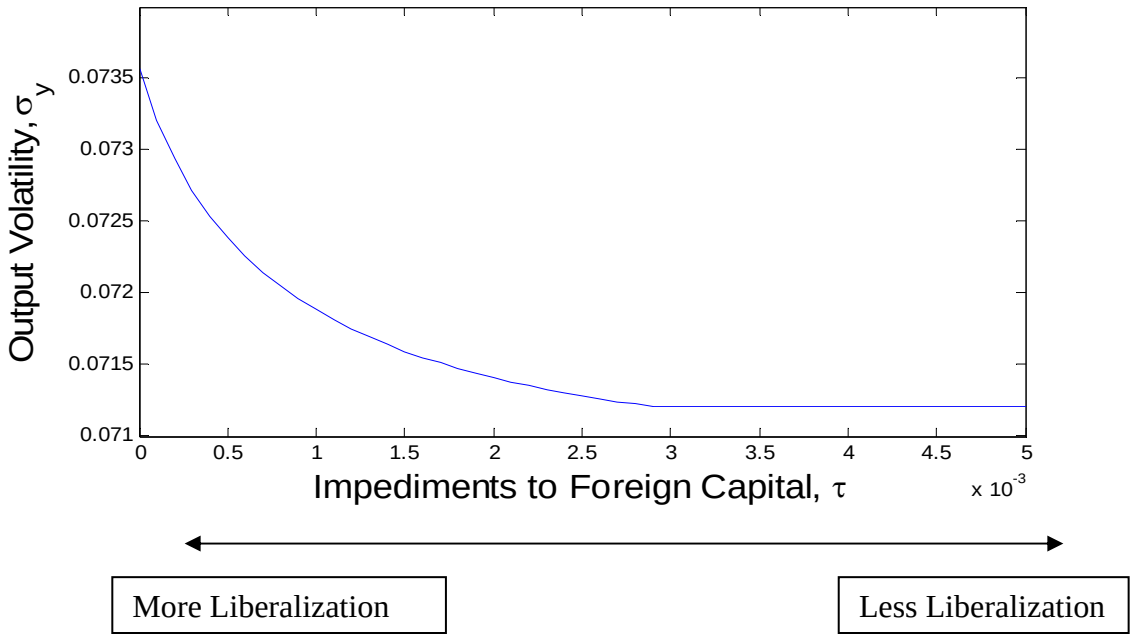
Note: The x-axis represents impediments to trade in foreign capital. As cross-country productivity correlations increase, two opposing effects affect production specialization.

Figure 3.10: Liberalization and Cross-Country Output Correlations



Note: The x-axis represents impediments to trade in foreign capital. On the right side, frictions in capital markets are high and the markets can be closed (no trade in financial assets takes place). As financial liberalization increases, countries allocate more capital in the most productive sector, increasing specialization, which in turn reduces output correlations across countries.

Figure 3.11: Liberalization and Output Volatility



Note: The x-axis represents impediments to trade in foreign capital. On the right side, frictions in capital markets are high and the markets can be closed (no trade in financial assets takes place). As financial liberalization increases, countries allocate more capital in the most productive sector, increasing specialization, which in turn increase output volatility.