Industrial Specialization, Financial Integration and International Consumption Risk Sharing*

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March 2014

Abstract

Standard models of international asset trade lack mechanisms linking an economy’s financial openness and industrial specialization. This paper presents a simple model of global portfolio diversification in which a link between financial liberalization and production specialization emerges very naturally. Within that model, an economy that liberalizes its financial markets is able to share its consumption risks, which in turn would allow the country to take extra risks in production by specializing in the good it produces more efficiently, resulting in higher output volatility and less correlated business cycles. The model provides insights about the effects of cross-country and cross-sector productivity shock correlations on portfolio choice, industrial specialization and consumption smoothing. Output volatility will increase as countries get more financially open, whereas consumption smoothing will depend on both the actual level of financial openness and on the productivity shock correlations between sectors and between countries. Thus, substantial liberalization does not necessarily mean an improvement in consumption risk sharing and equilibrium portfolios can be biased towards domestic assets.

1 Introduction

This paper analyzes the effects of financial integration on the sectoral production mix of an open economy and its feedback on consumption smoothing. In the context of a two-country model, it

*I am thankful to Jonathan Heathcote, Behzad Diba, Mark Hugget, Jinhui Bai, Olena Mykhaylova, Ed Gamber, Felicia Ionescu, Sami Alpanda, Yamin Ahmad, Dan Cao, Ilenin Kondo, Patricia Jones and seminar participants at Hamilton College, Amherst College and Georgetown Center for Economic Research for valuable comments. I am grateful to Tiago Cavalcanti and an anonymous referee for valuable suggestions and comments. Any mistakes are my own.

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also examines the effects of different degrees of sectoral and cross-country productivity correlations on portfolio and production diversification. The main results are that higher integration induces greater production specialization; higher cross-sectoral productivity correlations reduce production diversification while increasing portfolio diversification; and higher cross-country productivity correlations reduce financial diversification. The effect of cross-country productivity correlations on production diversification is ambiguous and depends on the degree of financial frictions. When the productivity of the more efficient sector in the home country becomes more similar to the productivity of the less efficient sector in the foreign country two things can happen. For low levels of financial frictions, greater cross-country productivity correlations lead to greater production specialization along with greater home bias. For higher levels of financial frictions though, greater cross-country productivity correlations induce greater production diversification. In the model presented here, production diversification acts as a substitute for portfolio diversification to achieve risk sharing, at the expense of higher expected output.

This paper is related to several strands of literature. The basic theme goes back to Arrow’s (1971 p.137) observation that "the mere trading of risks, taken as given, is only part of the story and in many respects the less interesting part. The possibility of shifting risks, of insurance in the broadest sense, permits individuals to engage in risky activities that they would not otherwise undertake." This is illustrated in a two-country, two-sector Ricardian framework with linear technology and stochastic productivity shocks. A productive resource (labor, capital) can be allocated across sectors. Production uncertainty in the model ensures that countries maintain a diversified production profile rather than specialize according to their comparative advantage. As consumers engage in international financial transactions, ensuring part of their consumption, countries can engage in producing more of the good they are best at, i.e., they devote a higher share of their resource to the production of the good they have an advantage in. The pattern of specialization increases with liberalization, but it does not have to reach full specialization as the role of international financial markets is to share the risks efficiently, not eliminate them.

The idea that product diversification and portfolio diversification are substitutes so that features delivered by portfolio diversification can be reproduced to some extent by product diversification has been addressed and formalized before (Saint-Paul (1992), Obstfeld (1994)). The predictions
of the model presented here are consistent with these studies\(^1\), while offering certain advantages. First, the model allows for closed formed solutions and the correlation structure across countries and across sectors offers new insights in the literature. Second, the model presented here focuses on volatility as well as output growth, and third, it also allows for some interesting static comparisons on how productivity shock correlations affect the relationship between financial openness, portfolio choice, consumption smoothing and industrial specialization. Another related framework is Koren (2003) that explains the low level of international trade (compared to the predictions of the gravity model) in a *Ricardian* framework with productivity shocks. In this paper, financial markets are modeled differently. Consumers can purchase shares of foreign output subject to a proportional tax. Financial globalization will correspond to a decline in these transaction costs. Also, the framework does not require strict assumptions, like no country-specific shocks. Differently from Koren (2003), we can observe non-full specialization in this model even if financial markets are fully open and complete. In addition, the closed form solutions allow for a complete mapping of home bias and consumption smoothing measures on financial openness, without restricting the analysis only to cases of financial autarky, partial and complete financial markets.

The heterogeneity in production introduced in this model suggests substantial gains in welfare from financial integration. The literature on the welfare effects of financial globalization has attempted to measure the benefits in two different ways. An extensive literature has focused on the benefits in terms risk sharing. The results vary widely, ranging from a fraction of a percent for industrial economies to about six percent of lifetime consumption for emerging markets and developing countries.\(^2\) A new strand of literature, based on an influential paper from Gourinchas and Jeanne (2006), has focused on measuring the benefits of financial integration coming from the capital scarcity of developing countries. They find that the gains are less than one percent of lifetime consumption, but according to them international financial integration would have considerable welfare effects if it leads to increased productivity. Antunes and Cavalcanti (2012) study the benefits of financial integration in a capital scarce economy with heterogeneous agents that face borrowing constraints. They find that the benefits of financial integration can be around 5.4%.

\(^1\)Hnatkovska and Evans (2007) is another study that shows that the insurance provided by international markets allows countries to expand production in risky sectors, thereby increasing welfare while increasing output volatility.

\(^2\)See Athanasoulis and van Wincoop (2000) and Kose, Prasad, Rogoff and Wei (2003) for a discussion of this literature.
increase in permanent consumption. The model presented here suggests that financial openness would increase welfare through resource reallocation. Although the results are not directly comparable to the studies mentioned, as this paper abstains from dynamic considerations, back of the envelope calculations suggest that in this framework consumption gains could be considerable for an economy that switches from financial autarky to complete financial openness and they depend on the productivity differences between sectors.

A large number of papers have analyzed portfolio home bias, which is not predicted by standard models in international macroeconomics (Coeurdacier (2008), Heathcote and Perri (2013), Bhamra, Coeurdacier and Guibaud (2012), Coeurdacier, Kollman and Martin (2010) among others). The literature has focused mainly on costs of trade and international price movements to explain the high degree of domestic assets in the portfolio of households. The predictions of this model are in line with some of the empirical findings of the papers mentioned above. This paper finds that for small financial frictions, high cross-country productivity correlations would explain home bias. In that sense, it is the most similar to Bhamra, Coeurdacier and Guibaud (2012) that analyze a continuous-time, two-country endowment economy. Heathcote and Perri (2013) show empirical support that countries that are more open to trade are more financially diversified. In our framework, countries will specialize according to their comparative advantage and the increase in trade will be one-to-one with financial integration. Similar empirical findings are supported by Coeurdacier (2008), Collard, Dellas, Diba and Stockman (2007), Aviat and Coeurdacier (2007), Lane and Milesi-Ferretti (2008) and Portes and Rey (2005). Ju and Wei (2009) incorporate labor market rigidities in a Heckscher-Ohlin framework and show that capital can flow from a labor-abundant to a capital-abundant country when there is a positive productivity increase in the labor-intensive sector of the former. In addition to labor market rigidities, trade costs and costs to international capital flows work against comparative advantage in shaping specialization patterns. Jin (2012) develops a general equilibrium framework in which two opposing mechanisms operate. The first is the standard, or "convergence" mechanism, which channels capital toward the sector where the effective capital-labor ratio is lower. The second is the "composition" one, in which capital tends to flow toward

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3 Heathcote and Perri (2013) and Coeurdacier, Kollman and Martin (2010) emphasize the role of investment and capital accumulation in explaining home bias. This paper focuses only on the effects of productivity similarities on portfolio bias. It would be interesting future work to include investment and to compare results with these papers.

4 In their model, similar to the one in this paper, capital can adjust costlessly across sectors, and this dampens the need for cross-border flows.
economies that have become more specialized in capital-intensive sectors. The direction of capital flows depends on which one of the two effects dominates. Our model abstracts from Heckscher-Ohlin consideration and emphasizes the role of productivity shock correlations on the volume of financial flows and patterns of specialization.

Some of the work mentioned above (Heathcote and Perri (2013), Bhamra, Coeurdacier and Guibaud (2012)) and other works (Tesar (1993), Backus and Smith (1993), Benigno and Thoenissen (2008), Corsetti, Dedola and Leduc (2008)) focus on the role of international prices and nontradeable goods in explaining home bias and the lack of international risk sharing in the data (as compared to canonical models in open economy macro). They emphasize the role of terms of trade and the elasticity of substitution between tradeables and nontradeables in explaining the discrepancy between the predictions of the models and the data. This paper does not deal with dynamics, so a direct comparison is beyond its scope. In addition, to achieve a closed form solution the goods are assumed to be perfect substitutes. Still, some modifications to the model can give us some (indirect) insights about the role of international price movements. In a robustness exercise, the perfect substitutability of goods across countries assumption is relaxed. The analysis shows that this would have the same effect as an increase in the impediments to purchasing foreign capital, thus increasing home bias. This suggests that the model presented here is a good candidate to study further issues in international finance.

Cross-border financial flows have increased substantially over the last two decades offering possibilities for countries to diversify risks internationally. The empirical works of Kalemli-Ozcan, Sorensen and Yosha (2003), Imbs (2004) and Basile and Girardi (2009) show that risk sharing enhances specialization in production\textsuperscript{5} and the recent paper by Kalemli-Ozcan, Papaioannou and Peydro (2011) provides empirical evidence that a higher degree of financial integration is associated with less synchronized output cycles. This paper rationalizes on these findings and emphasizes a specific channel of how financial integration can enhance specialization (and trade), which to the best of our knowledge is missing in the theoretical literature up to date. The results show that output volatility increases, while output correlations across countries decrease.

The model can give insights on the effects of financial liberalization on consumption smoothing.

\textsuperscript{5}Heathcote and Perri (2013), Aviat and Coeurdacier (2007) and Collard, Dellas, Diba and Stockman (2007) show empirically a strong correlation between trade and financial integration across developed economies.
In this model substantial liberalization may not mean an improvement in consumption smoothing, consistent with the empirical literature in this area.\footnote{See Kose, Prasad and Terrones (2009) for a review.} There is a large empirical literature investigating the effects of financial liberalization on consumption smoothing that has concentrated on consumption based measures, mainly consumption-output correlations and cross-country consumption correlations.\footnote{The literature also looks at consumption volatility divided by output volatility. That can be calculated with this framework and the predictions are the same as with the other measures, so it has been excluded to avoid repetitions. Further details are available upon request.} The findings of the empirical literature are at best inconclusive. While some degree of consumption risk sharing has occurred within the group of industrial economies, the same cannot be concluded for emerging markets and developing countries, despite the surge in financial globalization in the past few decades. Several recent papers suggest that the following features are central to understanding the relationship between financial openness and consumption smoothing: (i) financial impediments (Lewis (1996) is a seminal reference paper), (ii) business cycle properties (Heathcote and Perri (2004)), Artis and Hoffmann (2008), and (iii) the relationship is most likely non-linear (Kose, Prasad and Terrones (2003), Islamaj (2012a)). The predictions of this model offer several insights in relation to these measures. First, the actual impediments to trading foreign capital and cross-country productivity correlations can help explain why empirical studies fail to find improvements in consumption smoothing as countries have become more financially liberalized. Small financial frictions would be enough in this model to bias both portfolio and consumption choices towards domestic assets and consumption goods.\footnote{Consistent with Cole and Obstfeld (1991).} Second, cross-country productivity correlations can deteriorate consumption based measures of risk sharing. Kose, Otrok and Whiteman (2003) provide evidence of a world business cycle, which suggests a high correlation of business cycles between countries. Heathcote and Perri (2004) argue that as business cycles become more correlated, the gains from diversifying risk become smaller, and this in return decreases the holding of foreign shares from consumers, causing a deterioration in consumption smoothing. The model presented here extends this idea by allowing to study the effects of financial liberalization on portfolio choice and consumption smoothing, while accounting for changes in productivity shock correlations not only between countries, but also between sectors. Third, this model shows that the relationships between liberalization and risk sharing, and productivity correlations and risk sharing are nonlinear. This nonlinearity is not merely a mathematical fact, but has important
implications about consumption smoothing. This paper presents a general equilibrium framework that solves for closed form solutions and can capture the nature of these nonlinearities.

The rest of the paper is organized as follow. Section 2 explains the model. In this economy, firms allocate resources, whereas consumers make portfolio choice decisions. Section 3 briefly describes the solution for the symmetric case. This simplification helps us focus on the roles of cross-country and cross-sector productivity shock correlation in specialization, volatility and business cycle comovements. The results are shown in Section 4 and can be summarized as follows. First, more liberalization increases specialization. Second, higher cross-sectoral productivity correlations increase financial diversification and industrial specialization. Finally, higher cross-country productivity correlations decrease portfolio diversification, while the effects on the sectoral mix of production are ambiguous. For low values of impediments to trading foreign capital, firms increase specialization, whereas for higher values of impediments there is more production diversification. Section 5 shows further robustness checks including volatility asymmetries and imperfect substitutability of consumption across countries. Some concluding remarks and suggestions about future work are offered in the last section.

2 The Model

This section introduces a simple framework to model the issues depicted in the introduction. Consider a two-country, two-good Ricardian world with uncertainty in production. A fixed amount of a particular resource (labor, capital) will be allocated in two sectors where one is more productive than the other. In the absence of international financial markets, risk sharing will be achieved through diversification. That is to say, the resource will be allocated to both sectors. The main purpose of financial markets in this model is to share risks, thereby allowing the smoothing of consumption across various "states of nature". By sharing consumption risks in the financial markets, countries can increase expected production by allocating more of the productive resource in the most productive sector. In turn, this will increase output volatility. The framework yields explicit closed-form solutions for volatility and other second moments. Cross-country and cross-sector productivity correlations, as well as impediments to trading foreign capital, will affect specialization, output volatility and consumption based measures of risk sharing.
At the beginning of the period, a representative consumer owns $k$ units of the productive resource (capital) and sells it at price $p$ to a representative firm that allocates it into two sectors, $a$ and $b$. Production in each sector depends on technology and on a stochastic productivity shock, and it is defined as $y_i(s) = z_i(s)A_i k_i$, for $i \in \{a, b\}$. $A_i$ represents the technology coefficient in sector $i$, $z_i(s)$ represents stochastic productivity in sector $i$, and $s \in \{1, 2, ..., S\}$ is the state of nature. Let’s assume that $z_i \sim i.i.d.(\mu_i, \sigma^2)$.\(^9\) The production is similar in the foreign country (*). Using the income from selling capital, the domestic (foreign) consumer buys shares, $\lambda_i$ and $\lambda^*_i$ ($\eta_i, \eta^*_i$), of each sector in the domestic and foreign country at prices $q_i$ and $q^*_i$, respectively, for $i \in \{a, b\}$. Foreign shares will be taxed by a fraction $\tau$, which will represent impediments to purchasing foreign capital. Once the consumer chooses her shares and capital is allocated across sectors, the state of nature is revealed and the domestic and foreign consumers consume their claims.

2.1 Domestic Firms’ Problem

The domestic firm allocates capital to maximize revenues minus costs. The costs in this case are just the value of the resources it purchases. Then, the firm chooses the resources ($k_i$) for sectors $a$ and $b$ to maximize:

$$\max_{k_i \geq 0} \left\{ \sum_i q_i - p(\sum_i k_i) \right\}, \quad \forall i \in \{a, b\}$$

(1)

where

$$q_i = E[Q(s)z_i(s) A_i k_i]$$

(2)

$$\sum_i k_i = k$$

$E[.]$ represents the expectations operator, and $Q(s)$ is the price of a unit of output in state $s^{10,11}$, and consumer’s portfolio choice, $\lambda_i$ and $\lambda^*_i$, is taken as given.

\(^9\)Note that $\sigma$ is assumed the same across sectors for convenience. Letting the volatilities in different sectors be different will not change the analysis, something we investigate further later in the paper.

\(^{10}\)Let’s assume that sectors $a$ and $b$ produce goods that are perfect substitutes to each other.

\(^{11}\)Note that an interior solution requires that $q_i = p k_i$, for $i \in \{a, b\}$, and in equilibrium $q_i$ represents the value of the firm.
First Order conditions (assuming interior solution):

\[ p = E[Q(s)z_i(s)A_i] , \quad \forall i \]  \hspace{1cm} (3)

which implies:

\[ E[Q(s)z_a(s)A_a] = E[Q(s)z_b(s)A_b] \]  \hspace{1cm} (4)

In equilibrium, price of output will equal marginal utility of consumption, so \( Q(s) = u'(c(s)) \):

\[ E[u'(c(s))z_a(s)A_a] = E[u'(c(s))z_b(s)A_b] \]  \hspace{1cm} (5)

\[ A_aE[u'(c)z_a(s)] = A_bE[u'(c)z_b(s)] \]

\[ A_a[cov(u'(c), z_a(s)) + E[u'(c)]E[z_a(s)]] = A_b[cov(u'(c), z_b(s)) + E[u'(c)]E[z_b(s)]] \]

Assuming the utility is exponential \( u(c(s)) = \frac{1}{\theta}e^{-\theta c(s)} \), where \( c(s) \) is consumption at state \( s \), and goods \( a \) and \( b \) are perfect substitutes, and that productivity shocks \( z(s) \) come from a multivariate normal distribution, and applying Stein’s Lemma\(^{12}\), the equation above becomes\(^{13}\):

\[ (-\theta cov(c, z_a) + \mu_a)A_a = (-\theta cov(c, z_b) + \mu_b)A_b \]  \hspace{1cm} (6)

where \( \mu_a \) and \( \mu_b \) are the means of the productivity shock in each sector.

The framework yields closed-form solutions for portfolio holdings, the allocation of resources, output volatility, output correlations and measures of consumption smoothing. Those solutions will be affected by six (cross-country and cross-sector) productivity shock correlations. Let’s denote the cross-sector correlations as \( \rho_{ab} \) and \( \rho_{a^*b^*} \) (sectors \( a \) and \( b \) in each country are correlated), the cross-country ones as \( \rho_{aa^*} \) and \( \rho_{bb^*} \) (sector \( a \) in home country is correlated with sector \( a \) in the foreign country, and sector \( b \) in home country is correlated with sector \( b \) in the foreign country), and, \( \rho_{ab^*} \) and \( \rho_{a^*b} \) (sector \( a \) in home country is correlated with sector \( b \) in the foreign country, and sector \( b \) in home country is correlated with sector \( a \) in the foreign country).\(^{14}\) This paper studies the effects of these correlations on volatility and allocation of capital across sectors.

\(^{12}\) Stein’s lemma states that if \( a \) and \( b \) are jointly bi-variate normal variables and \( g \) is a differentiable function, \( cov(g(a), b) = E(g'(a))cov(a, b) \)

\(^{13}\) See the Appendix for more details.

\(^{14}\) The equations are analogous for the foreign firm.
2.2 Consumers’ Problem

Domestic (foreign) consumers make their portfolio choices, \( \lambda_i, \lambda_i^*(\eta_i^*, \eta_i) \leq 1 \)\(^{15} \), \( i \in \{a, b\} \), that maximize their expected utility taking the capital allocations, \( k_i \) and \( k_i^* \), as given from the firms’ problem:

\[
\max_{\lambda_i, \lambda_i^* \in [0, 1]} E[u(c(s))] \tag{7}
\]

given \( k_i \) (and \( k_i^* \)), s.t.

\[
\sum (q_i \lambda_i) + \frac{1}{(1 - \tau)} \sum (q_i^* \lambda_i^*) = p k
\]

\[
c(s) = \sum \lambda_i y_i(s) + \sum \lambda_i^* y_i^*(s) \tag{8}
\]

\( \tau \in [0, 1] \) represents an iceberg cost when purchasing foreign shares\(^{16} \). When \( \tau \rightarrow 0 \), the costs are really low and it can be thought of as representing a fully integrated economy. If \( \tau \rightarrow 1 \), then the costs to trading foreign shares are really high, consumers will not buy them, and this can represent a situation of financial autarky. The problem is analogous for the foreign consumer.

Three independent equations come out of the domestic consumer’s problem (See the Appendix for more details). Analogously, three more equations are derived from the foreign consumer and 4 more equations represent the stock market clearance conditions:

\[
\lambda_a(s) + \eta_a(s) = 1 \quad \lambda_a^*(s) + \eta_a^*(s) = 1
\]

\[
\lambda_b(s) + \eta_b(s) = 1 \quad \lambda_b^*(s) + \eta_b^*(s) = 1 \tag{9}
\]

From these 10 equations, one can pin down 8 portfolio choices (for the domestic and foreign consumers) and prices \( p \) and \( p^* \) at which consumers sell their productive resource to firms.

2.3 Definition of Equilibrium

Denote \( s \in (1, 2, ..., S) \), for \( i \in \{a, b\} \)

\(^{15}\)We do not allow agents to go short in foreign shares, since this would allow agents to increase expected consumption. Agents may go short in domestic shares but opt not to in equilibrium.

\(^{16}\)Here we assume that tax revenues are wasted but results are similar if we assume that revenues are rebated lump sum to consumers.
An equilibrium is a set of quantities $\lambda_i(s), \lambda_i^*(s), \eta_i(s), \eta_i(s)^*$, prices $p, p^*$, and productivity shocks $z_i(s), z_i^*(s_i^*)$ which satisfy the following conditions:

1. Market clearing condition for goods:

$$c_i(s) + c_i^*(s) = y_i(s) + y_i^*(s) = k_i A_i z_i(s) + k_i^* A_i^* z_i^*(s), \quad \forall i$$

2. Market clearing condition for stocks:

$$\lambda_i(s) + \eta_i(s) = 1; \quad \lambda_i^*(s) + \eta_i^*(s) = 1, \quad \forall i$$

### 3 Symmetric case

Consider the case when both countries are symmetric. In that case productivity in sector $a$ at Home is the same as the productivity in sector $b$ in Foreign.

$$A_a = A_b^*$$

$$A_b = A_a^*$$

$$p = p^*$$

The expected values and volatilities of productivity shocks are the same in all sectors in both countries. As a result, the choice of shares will be symmetric. Foreign country is the mirror image of Home, so whatever Home’s choice for sector $a$ will be Foreign’s decision for sector $b$, and vice versa. In that case,

$$\lambda_a = 1 - \eta_a = \eta_b^* = 1 - \lambda_b^*$$

$$\lambda_b = 1 - \eta_b = \eta_a^* = 1 - \lambda_a^*$$

$$k_a = k - k_b = k_b^* = k^* - k_a^*$$

In this case, the equilibrium $\lambda_i$ can be pinned down by the first two independent equations coming out of the FOC’s (See Appendix for the FOC).
This case is interesting because it serves as a starting point to investigate the role of financial markets and productivity shock correlations on resource allocation and portfolio choice. The next section will show the portfolio holdings, resource allocations and consumption smoothing for each value of $\tau$ for the symmetric case. More sensitivity analysis will show what happens when cross-country and cross-sectoral productivity correlations change, as well as what happens for small deviations from this simplified case.

4 Results

4.1 Implications of the Model

This section discusses some theoretical implications of this model. First, it looks at portfolio holdings and resource allocation across sectors. The main message is that financial openness increases industrial specialization, and economies may not completely specialize even in the case when there is an absolute advantage in the production of one good. Next, it analyzes volatility, output correlations and consumption based measures of risk sharing and discusses what happens as impediments to foreign capital decline. Then, it looks at what happens to portfolio holdings as the productivity correlations change. The model is basic, yet rich enough to control for changes in cross-country and cross-sectoral productivity correlations. We find that the more similar the productivity processes between countries, the higher the home bias. The intuition is that, as the probability of the same (good/bad) shock occurring in the two countries at the same time increases, the gains from sharing risk would be smaller. On the other hand, the higher the correlation of productivity processes across sectors in each country, the lower the home bias and the higher the incentives of individuals to share income risks with the rest of the world.

\footnote{Relaxing this assumption does not change the analysis. We believe that this simple case is enough to emphasize the role of financial globalization and (cross-country/cross-sector) productivity shock correlations on industrial specialization, volatility and measures of consumption risk sharing.}

\footnote{The story is consistent for comparative advantages in production.}
4.1.1 Financial Liberalization, Productivity Shock Correlations and Industrial Specialization

This model provides a framework to relate financial liberalization, portfolio holdings and specialization. Most importantly, in this framework, it is possible to have absolute advantages in production, but still not fully specialize accordingly although there are no trade frictions. As explained above, individuals in each country face uncertainty in production in both sectors. If they specialize in only one good, there exists a possibility that they will be faced with a bad shock and not be able to meet their consumption needs. If there were financial autarky, consumers would share this risk internally by producing both goods. When financial markets are open, consumers engage in trading claims to output with the other country. In this case, the consumers are provided with extra insurance and this allows them to increase risks in production. But, financial markets serve only to share risks efficiently, not to eliminate them (even when markets are complete). Thus, production risks are still present, and as a result we do not see full specialization.

Figures la-b, shows what happens to portfolio holdings as impediments to trade in foreign capital decrease. In this example, $A_a = 1.01$, $A_b = 1$, $\mu_i = 2, \forall i$, $\sigma = 0.2$ and the risk aversion parameter $\theta = 1.75$. $\rho = 0$ corresponds to the case when productivity shocks across countries and across sectors are uncorrelated.\(^{19}\) Moving left on the horizontal axis represents less frictions in buying foreign assets and can be interpreted as financial liberalization. When the costs are sufficiently high, the constraint that agents cannot go short in foreign stocks is always binding and there is complete home bias ($\lambda_i = 1$).\(^{20}\) As financial frictions decrease, consumers hold fewer domestic assets. Purchases of foreign assets also increase as financial openness increases (not shown in the figure). Full liberalization corresponds to a home bias of $1/2$.\(^{21}\) The results are similar for holdings of shares in sector $b$ and foreign holdings of shares for both sectors are 0 for restrictive impediments, whereas home bias is equal to $1/2$ for fully liberalized markets.

What happens to portfolio holdings if productivity shock correlations between countries and/or

\(^{19}\) Islamaj (2012a) measures cross-country productivity correlations and finds that $\rho = 0.07$ for developed countries, and $\rho = 0.2$ for emerging markets and developing economies.

\(^{20}\) Notice that even small frictions are enough to shut down the international markets. This is in line with other findings in this literature (Cole and Obstfeld (1991), Tesar and Werner (1995)). The high volume and turnover of trade in foreign assets also suggests that these costs are small.

\(^{21}\) This corresponds to the result in Heathcote and Perri (2004) for the one-good endowment economy.
sectors are greater than zero? Figure 1a compares, for each value of $\tau$, the case of uncorrelated shocks (solid (blue) line) with the case of positively correlated sectoral shocks within each country (dotted (green) line), i.e., $\rho_{ab} = \rho_{a^*b^*} = 0.2$, while $\rho_{aa^*} = \rho_{bb^*} = \rho_{ab^*} = \rho_{a^*b} = 0$. In this case, consumers diversify more across countries and less across sectors. The intuition is that the probability of both sectors being subject to a bad shock has gone up, so now there are fewer incentives to share risks by diversifying production across sectors. For intermediate values of the cost parameter, an increase in the correlation of the shocks decreases the marginal benefit of sectoral diversification and thus, holding constant the marginal cost $\tau$, leads to an increase in international diversification. Figure 1b compares the home bias between the uncorrelated shock scenario (solid line) and the case when cross-country productivity shocks are positively correlated, $\rho_{aa^*} = \rho_{bb^*} = 0.2$, while the other correlations are zero. In this case, correlated cross-country productivity processes provide fewer incentives to diversify risks by purchasing foreign assets. As a result, home bias deteriorates.\footnote{The result is similar to the one good endowment economy in Heathcote and Perri (2004).}

Figure 2 shows what happens to the productive resource allocation in sector $a$ as financial restrictions decrease. As mentioned, for high values of $\tau$, which correspond to closed financial markets, individuals in each country engage in the production of both goods in order to avoid risks in production.\footnote{Notice that more of the productive resource is allocated to the most productive sector $a$.} As impediments to trade in foreign capital decrease and consumers buy shares of foreign capital (solid line in Figure 2), consumers in each country enjoy some extra insurance. As a result, more resources are allocated to the production of the most efficient good (at the expense of the other less efficient good). The figure shows what happens to resource allocation in sector $a$ in the domestic country (resource allocation in sector $b$ is just $k$ (normalized to 1) minus resource supply in sector $a$). As expected, although there is some specialization (which leads to trade), the countries do not fully specialize because there still exist some production risks that are not fully insured. The results are analogous for the foreign country (note that in the foreign country sector $b$ is more efficient).

Figure 2 also shows the effect of cross-sector productivity correlations ($\rho_{ab} = \rho_{a^*b^*} = 0.2$) on resource allocation (dotted line). For each $\tau$, the degree of specialization is higher than the uncorrelated productivity case (more resources allocated to the more productive sector $a$). The
intuition is that in this case the probability of both sectors having the same bad shock is higher, so it makes sense to focus more resources in the sector with the highest expected returns. Figure 3 tracks changes to output volatility as impediments to foreign capital decrease and shows that output becomes more volatile as specialization increases.

Figures 4a-b show what happens to industrial specialization as \textit{cross-country} productivity correlations increase. An interesting result is that an increase in cross-country productivity correlations can have mixed effects on production diversification, depending on financial frictions and the exact nature of the productivity shocks. To understand the nature of these effects better, we differentiate between correlations between same sectors in different countries and correlations between different sectors in different countries. More specifically, first we look at what happens when $\rho_{ab} = \rho_{a^*b} = 0.2$ and then the case when $\rho_{aa^*} = \rho_{bb^*} = 0.2$, while fixing the other correlations equal to zero. Figure 4a shows that as the productivity processes of the more productive sectors across countries become more similar ($\rho_{ab^*} = \rho_{a^*b} = 0.2$) the incentives to share risks across countries decline and less resources are allocated into the more productive sector $a$. Thus, countries diversify production to share risks in response to an increase in the correlation of productivity processes in the most productive sectors across countries.

Figure 4b shows the most novel implication of the paper. It compares the uncorrelated shocks scenario with the case of positive cross-country (same sector) productivity correlations ($\rho_{aa^*} = \rho_{bb^*} = 0.2$). This figure reveals some interesting insights. For low levels of financial frictions, higher correlations between sectors $a$ and $a^*$, and $b$ and $b^*$, lead to more specialization, not less. When financial markets are fully open, Home consumer owns half of the most productive foreign sector $b^*$, which is now more correlated with the least productive home sector $b$. Home country increases production in sector $a$ and, analogously, Foreign country increases production in sector $b^*$. This allows the countries to expect higher returns from the most productive sector abroad and creates fewer incentives to diversify production at home. As a result, countries shy away from the least productive home sector (which can be substituted for higher expected returns from the most productive foreign sector that is now more correlated to this sector) and increase resource allocation in their most productive sector. For high levels of financial frictions, as countries become

\footnote{Note that sector $a$ is the most productive sector in Home Country and sector $a^*$ is the least productive sector in Foreign Country.}
more similar, there are fewer incentives to diversify abroad and this will lead to more diversification across sectors at home (resources allocated to sector $a$ decline). Thus, financial frictions introduce home bias in portfolios and there exists a threshold beyond which the marginal benefit of sharing risks through production in sector $b$ becomes higher than the marginal benefits from owning shares in $b^*$.

As expected, output becomes less correlated across countries, and domestically more volatile. Figure 5 graphs output correlations in this framework and shows that business cycles become less correlated as countries become more financially integrated\(^\text{25}\), in line with recent empirical studies mentioned earlier. In this graph $\rho_{aa^*} = \rho_{bb^*} = 0.2$, while the other correlations are zero. When productivity is uncorrelated across countries and sectors, the output correlation is trivially zero (See the Appendix for details.). Figure 6 shows what happens to output volatility as the countries become more financially liberalized. As restrictions decrease, and more resources are allocated to the more productive sector, output becomes more volatile. The solid line represents the case when productivity is uncorrelated across countries and the dotted line shows the same relationship when $\rho_{aa^*} = \rho_{bb^*} = 0.2$. The effect of cross-country productivity correlations on output volatility depends on the level of financial impediments. The reason is explained in the previous paragraph and it can be noticed that it depends directly on the proportion of resources allocated to the more productive sector.

Figures 7 and 8 show a 3-D representation of the interdependencies of home bias and resource allocation for arrays of $\tau$ and cross-country productivity shock correlations $\rho$ ($\rho_{aa^*} = \rho_{bb^*}$). The results are the same as discussed above, suggesting that they hold for a wide range of choices of $\tau$ and $\rho$.

### 4.1.2 Financial Liberalization, Productivity Shock Correlation and Consumption Risk Sharing

The literature on consumption risk sharing falls into two broad categories. The first strand emphasizes that in complete financial markets, marginal utility growth should be equated across countries so that consumption growth rates should be highly correlated. This correlation-based approach predicts that, absent financial market frictions, not only should consumption be highly correlated

\(^{25}\)Remember that we are talking about the symmetric case.
across countries, but it also should be more correlated than output. A second strand of the empiri-
cal consumption risk sharing literature has focused on regression-based measures. These studies
emphasize an alternative prediction of the complete markets model: fluctuation in relative marginal
utility growth should be independent of idiosyncratic risk (as measured by relative output growth
rates). Therefore, the coefficient of a regression of relative consumption growth on relative output
growth should be low under high degrees of financial liberalization. The acknowledgement that
real financial markets are likely to be incomplete has led researchers to adopt a more pragmatic
approach in applied work, and argue that the coefficient of this regression can be of interest in
itself and that it should be interpreted as a measure of the deviation from the complete markets
outcome.\textsuperscript{26}

All these available measures of consumption risk sharing are associated with potential advan-
tages and disadvantages. So far, the literature has not assumed that any one measure is better
than the others, but, instead, it has used them interchangeably. The derived closed-form solutions
for consumption correlations and consumption-output correlations allow us to investigate what
happens to these measures of consumption smoothing as countries get more financially liberalized
(See Appendix for details). The results give insights about important differences across measures
of consumption smoothing and about the role of cross-country and cross-sector productivity shock
correlations. Figure 9a shows what happens to the measures of consumption smoothing as restric-
tions to trade in capital shares decrease. The dotted thick blue line shows $corr(c, c^*)$. Standard
theories predict that in financially open environments consumption should be highly correlated
across countries. The solid red line shows $corr(c, y)$. For both measures of consumption smoothing,
more liberalization means better smoothing, everything else equal. It should be noted that these
relationships are highly nonlinear and the actual level of impediments to foreign capital matters.
For example, for high levels of $\tau$, as impediments to trade in foreign capital decline, there is little
change in $corr(c, y)$ (solid line). Only for low $\tau$ do we observe an increase in consumption smoothing
as restrictions to trading capital shares decrease.

What happens if productivity shock correlations between countries and/or sectors are greater

\textsuperscript{26}Another prediction in the theoretical literature of consumption risk sharing is that volatility of consumption should
be low in financially open settings (some recent studies have been looking at the ratio of consumption volatility and
output volatility to better control for output crises).
than zero? Figure 9b shows how these measures of consumption smoothing differ when productivity

correlations across countries are different from zero, but cross-sector productivity shock correlations

are zero, i.e., \( \rho_{aa^*} = \rho_{bb^*} = 0.2 \) and \( \rho_{ab} = \rho_{a^*b^*} = 0 \). Again, the solid line shows the case

when all productivity correlations are zero and the dotted line shows the case when productivity
correlations across countries are positive. In this case the home bias is higher and consumption

smoothing deteriorates. For a fixed intermediate value of \( \tau \), \( corr(c,y) \) is higher and \( corr(c,c^*) \)
is lower. The intuition would be that there are fewer incentives to diversify risks by purchasing

assets of the foreign country since the probability of both countries experiencing a negative shock

is higher. Also, we notice that the effect is higher for \( corr(c,y) \), suggesting that the magnitudes of

the effects of cross-country productivity correlations on these measures of consumption risk sharing

are not the same. The exact opposite is true when cross-sector productivity correlations increase

(consumption smoothing measures improve as it makes more sense to diversify internationally when

sectors within a country are subject to similar shocks). Figure 9c confirms these results. In this

case, it is also noticeable that \( corr(c,c^*) \) responds more than \( corr(c,y) \) to the same increase in

cross-sector productivity correlations.

It should be clear from the analysis so far that an increase in cross-country productivity shocks

and an increase in cross-sector productivity shocks within countries have opposing effects on home

bias and measures of consumption risk sharing. Which effect dominates? Figure 9d shows what

happens, for each \( \tau \), when all productivity correlations are zero (solid line) and when both cross-

country and cross-sectoral productivity correlations are positive, i.e., \( \rho_{aa^*} = \rho_{bb^*} = \rho_{ab} = \rho_{a^*b^*} = 0.2 \). In all cases the curve jumps upwards. But we read these measures differently. We say

that a country is better able to smooth consumption if it experiences a higher \( corr(c,c^*) \), and a

lower \( corr(c,y) \). We can see from the figure that consumption risk sharing has deteriorated if our

measures of consumption smoothing is \( corr(c,y) \), but, at the same time, it has improved if we focus

on \( corr(c,c^*) \). So far the literature has treated all these measures as equivalent, but the analysis

here shows that they can respond differently to different changes in productivity shock correlations.

The framework presented here supports the idea that cross-country and cross-sector productivity

shocks are important determinants of portfolio allocations, resource allocations and measures of

consumption risk sharing (or other second moments). More empirical work should be done in

measuring and identifying these correlations. Figures 10a and 10b show a 3-D representation
of financial impediments, cross-country productivity correlations and measures of consumption smoothing. The figures confirm the results discussed above.

_Ceteris paribus_, there exists a nonincreasing relationship between liberalization and portfolio choice, as well as between openness and consumption risk sharing. This is what standard theories suggest, but there are a few caveats to these results. First, these relationships are complex, nonlinear ones and depend on the actual level of impediments in trading foreign capital. For example, for high enough levels of financial market frictions, we can see little or no change in portfolio choice, resource allocation and consumption risk sharing as countries get more liberalized. Only for low enough levels of financial frictions do we see changes in these variables as markets liberalize further. Second, everything else equal, higher cross-country (cross-sector) productivity correlations imply higher (lower) home bias. Third, if a country has liberalized, but at the same time its productivity correlations with the rest of the world have increased, we may see an increase in home bias.\(^{27}\)

The results are robust to variations in the preference parameter, \(\theta\), and to changes in the mean and volatility of the productivity shocks. The basic results of the model still hold if the symmetry assumption is dropped, which will be useful in quantitative evaluations of these results. The model abstains from any dynamic considerations and it is silent on the empirical findings that cross-country output correlations are higher than cross-country consumption correlations, in contradiction with standard open macroeconomic models (Backus, Kehoe and Kydland 1995). It would be interesting to investigate these and other issues in a richer model under a dynamic setting, while carefully considering the role of productivity shock correlations.

As mentioned in the introduction, a large literature exists that tries to measure the welfare benefits of financial integration. Most of that literature has focused on the welfare benefits which arise from increased risk sharing and reports mixed results about the quantitative gains coming from financial globalization. Gourinchas and Jeanne (2006) emphasize the welfare gains that come through capital accumulation and productivity increases as a result of financial flows. In a heterogeneous agent framework with stochastic labor productivity shocks, Antunes and Cavalcanti (2012) show that these gains can indeed be considerable. The _static_ framework presented here, while not directly comparable to this literature and in no way a substitute for those studies, suggests con-

\(^{27}\) This is in line with previous studies, such as as Coeurdacier (2009), Heathcote and Perri (2013), Hnatkovska (2010).
siderable (indirect) benefits of risk sharing through gains in labor productivity, which arise from resource reallocation when the country liberalizes. The shift of productive resources from sector \( a \) to sector \( b \) when the country moves from financial autarky to complete financial integration would increase expected consumption, and, depending on the extent of productivity differences and the structural change in the economy, the welfare benefits can be considerable.\(^{28}\) Figure 11a shows the change in expected consumption when moving from financial autarky to complete financial openness. In this case, sector \( a \) is 1% more productive than sector \( b \), and \( k \) is normalized to 1. Lifetime expected consumption increases by, approximately, 0.3%. If sector \( a \) is 2% more productive than sector \( b \), then lifetime expected consumption increases by 0.8% (Figure 11b). If sector \( a \) is 5% more productive, then expected consumption increase by 1.9% (not shown in the figures, but available upon request).\(^{29}\) A quantitative exercise in a dynamic setting, that involves capital accumulation and can be comparable to other findings on the subject, remains the subject of future research.

5 Robustness Exercises

5.1 Allowing for Asymmetric Volatility of Productivity Shocks.

The results shown so far were derived under some simplifying assumptions to preserve tractability and to clarify the understanding of the role of productivity correlations (cross-sector and cross-country) on portfolio choice, industrial specialization and consumption smoothing. While the assumptions are helpful for the analysis, they can be relaxed and the model can be solved for the general case. In this section we relax two such assumptions and comment on the implications. First, we relax the assumption of symmetric volatility shocks across sectors and across countries and study what happens to portfolio choice, industrial specialization and consumption smoothing in each case.

\(^{28}\)Assume \( E[c(s)]^{FA} \) and \( k_{FA}^{A} \) denote expected consumption and capital in sector \( a \) under financial autarky, and \( E[c(s)]^{FO} \) and \( k_{FO}^{A} \) denote expected consumption and capital in sector \( a \) under complete financial openness. Also, assume that the mean of productivity shocks is the same in both sectors and equal to \( \mu \). Then, under financial autarky, \( E[c(s)]^{FA} = k_{FA}^{A}A_{a}\mu + (k - k_{FA}^{A})A_{b}\mu \). For the symmetric case, when \( \tau = 0 \), \( E[c(s)]^{FO} = k_{FO}^{A}A_{a}\mu + (k - k_{FO}^{A})A_{b}\mu \) (remember that when \( \tau = 0 \), \( \lambda_{a} = \lambda_{b} = \lambda_{a*} = \lambda_{b*} = 1/2 \)). Since, \( k_{FO}^{A} > k_{FA}^{A} \) (Figure 2), then

\[ E[c(s)]^{FO} > E[c(s)]^{FA} \]

and, depending on the extent of structural change and the ratio \( \frac{A_{a}}{A_{b}} \), the benefits may be nontrivial.

\(^{29}\)Note that the ratio of the mean and variance of output used in these exercises correspond to the ones for all countries in PWT 8.0 (volatility is about 10% of the output mean). Productivity correlations are all equal to zero. Expected consumption benefits will vary as the parameters of the model change.
We distinguish between two cases. First, we only allow the volatility of sector \( a \) at Home to vary. Next, we allow the volatility of both sectors at home to vary from those of foreign. These changes have some interesting implications.

Figure 12 shows the response to portfolio holdings in sector \( a \) at home as the productivity in sector \( a \) (only) increases. In the figures, volatility increases from \( \sigma_i = 0.2 \) to \( \sigma_i = 0.22 \), where \( i \) represents sector \( a \) or \( b \). When sector \( a \) becomes riskier, consumers respond by holding fewer stocks of that sector. At the same time, they hold fewer sector \( b \) stocks at home for each level of financial frictions (not shown here to preserve space). Thus, as sector \( a \) at home becomes more volatile, consumers are more likely to smooth shocks in the financial markets. At the same time, production in sector \( a \) decreases and more resources shift from sector \( a \) to sector \( b \) (Figure 13).

Figure 14 shows what happens to portfolio choice when both sectors at home become more volatile (\( \sigma_a = \sigma_b = 0.22 \)). As expected, representative agents respond to this increase in volatility by engaging in more asset trade with the foreign country. Figure 15 shows what happens to the productive resource allocation in sector \( a \) as financial restrictions decline in the symmetric case and in the case when the volatility of productivity shocks is higher in both sectors in Home country, increasing production risks. The results are mixed and depend on the level of financial frictions. For \( \tau = 0 \) as the volatilities in both sectors increase, the planner/firm moves some resources from sector \( a \) to sector \( b \) in an effort to share some of the increased risks via production diversification. For higher financial frictions, there is less specialization via financial markets and more specialization through production diversification. Thus, there’s a threshold, \( \tau \), above which the country would respond to an increase in the volatility of both sectors by taking in more risks in sector \( a \). To understand this result better, remember that Home consumer holds fewer shares of the foreign sectors \( a^* \) and \( b^* \) when financial frictions are introduced. In that case, the marginal expected utility from consuming one more unit is higher than the marginal (dis)utility from increased production risks (remember from last section that consumption is lower for higher \( \tau \)).

Figures 16a-b show what happens to measures of consumption smoothing as the volatility of shocks at home increases. Figure 16a represents the case when only volatility in sector \( a \) increases and Figure 16b shows the scenario when volatilities increase both in sector \( a \) and in sector \( b \). The results confirm the problems emphasized above about the measures of consumption risk sharing and how they are used in the empirical literature. In all cases the curve jumps upwards. But
we read these measures differently. We say that a country is better able to smooth consumption if it experiences a higher \( \text{corr}(c, c^*) \), and a lower \( \text{corr}(c, y) \). We can see from the figures that consumption risk sharing has deteriorated if our measures of consumption smoothing is \( \text{corr}(c, y) \), but, at the same time, it has improved if we focus on \( \text{corr}(cc^*) \). Thus, changes in volatilities of productivity shocks affects different measures of risk sharing in different ways.

5.2 Imperfect Substitutability between Domestic and Foreign-Produced Goods

The literature has also emphasized the role of international price movements in portfolio choices. There is a large literature on this topic (as described in the Introduction) that has emphasized the role of terms of trade movements in explaining the consumption-exchange rate anomaly (as described by Backus and Smith (1993)) and portfolio home bias. These papers also, in various ways, account for the imperfect substitutability between tradeable and non-tradeable goods, and between domestic and foreign-produced traded goods. In the model presented here, foreign and domestic goods are perfect substitutes (as are sector \( a \) and sector \( b \) goods). We can depart from this last assumption without the model suffering in tractability. Although the static nature of the framework presented here is not directly comparable with the literature mentioned (and it is beyond the scope of this paper to do dynamic comparisons with some of the other papers), departing from this last assumption can give us some intuition as to how the imperfect substitutability between domestic and foreign produced goods would affect portfolio home bias in a more enriched version of this model that would allow for dynamic considerations. The answer is supportive of the previous literature showing that when domestic and foreign goods are imperfect substitutes, then portfolio home bias increases. The first order conditions show that it has the same effect as an increase in financial frictions.

Consider a consumption function like the one below:

\[
u(c(s)) = -\frac{1}{\theta} e^{-\theta[c^H(s)] + \xi c^F(s)}
\]  

(12)

where,

\[
c^H(s) = \lambda_a k_a A_a z_a(s) + \lambda_b k_b A_b z_b(s)
\]

(13)
\[ c^F(s) = \lambda^*_a k^*_a A^*_a z^*_a(s) + \lambda^*_b k^*_b A^*_b z^*_b(s) \]  

(14)

\( \xi < 1 \) shows that consumption of foreign-produced goods is not one-to-one substitutable for the consumption of domestic produced goods.

Consumers will maximize utility:

\[
\max_{\lambda_a, \lambda_b, \lambda^*_a, \lambda^*_b \in [0,1]} \sum_s \pi(s)u(c(s))
\]

(15)

s.t.

\[
q_a \lambda_a + q_b \lambda_b + \frac{1}{1 - \tau} (q^*_a \lambda^*_a + q^*_b \lambda^*_b) = pk
\]

\[
c(s) = \lambda_a y_a(s) + \lambda_b y_b(s) + \xi (\lambda^*_a y^*_a(s) + \lambda^*_b y^*_b(s))
\]

(16)

Following the same steps as in the Appendix we can show that the effect of \( \xi \) on this economy is similar to the effect of a higher tax \( \tau \).

\[
(-\theta \text{cov}(c, z_a) + \mu_a)\lambda_a A_a = (1 - \tau)\xi(-\theta \text{cov}(c, z^*_a) + \mu_a)\lambda^*_a A^*_a
\]

(17)

This represents the second equation in the FOC conditions and relates marginal utility from domestic shares with marginal utility from foreign shares. The effect is the same as a higher tax rate, and thus we expect a deterioration in portfolio home bias. Figures 17 and 18 confirm these results. Although the results confirm the main findings of the literature, this example is intended to give some intuition about the role imperfect substitutability of domestic and foreign consumption on home bias and it is not quantitatively comparable to previous studies. Adding dynamics of investment and international price movements in a two-country, two-sector model like the one presented here remains an objective for future work.
This paper develops a two-country, two-good (Ricardian) framework with uncertainty in production that relates financial globalization with industrial specialization, output volatility and consumption based measure of risk sharing. Each country produces both goods with different technologies that make one sector more productive than the other. A productive resource can be allocated across sectors. Each sector is subject to stochastic technology shocks and financial liberalization is measured as a tax on purchasing foreign capital. Under financial autarky, risks are diversified by spreading the productive resource across sectors. As countries become more financially liberalized, they benefit from risk sharing opportunities with the rest of the world, and this allows them to take more risks in production, allocating more resources to the more productive sector. This paper provides closed-form solutions for the equilibrium values of resource allocation, output volatility, output correlations and consumption based measures of risk sharing. The effects of cross-country and cross-sector productivity correlations on these measures is analyzed.

This setup has certain advantages that make it a good candidate for analyzing business cycle fluctuations. First, it is a general equilibrium framework that links financial liberalization and industrial specialization. Second, as productivity is uncertain, investors wish to maintain a diversified industrial structure rather than (fully) specializing according to their comparative advantage. Third, it incorporates international financial market frictions that can limit portfolio diversification. As these impediments decline, representing more financial openness, the framework yields closed-form solutions for output volatility and other business cycle indicators, such as output and consumption correlations. Finally, the framework incorporates cross-country and cross-sector productivity shock correlations that affect specialization, volatility and measures of consumption smoothing.
7 References

References


8 Appendix: Solutions and Definitions

8.1 Consumers’ Problem

Consumers make their portfolio choices that maximize their expected utility taking the capital allocation as given from the firms’ problem:

$$\max_{\lambda_a, \lambda_b, \lambda_a^*, \lambda_b^* \in [0,1]} \sum_s \pi(s)u(c(s))$$

given $k_a$ (and $k^*_a$)

s.t.

$$q_a\lambda_a + q_b\lambda_b + \frac{1}{(1-\tau)}(q^*_a\lambda^*_a + q^*_b\lambda^*_b) = pk$$

$$c(s) = \lambda_a y_a(s) + \lambda_b y_b(s) + \lambda^*_a y^*_a(s) + \lambda^*_b y^*_b(s)$$
First order conditions (assuming all stock purchases interior):

\[-q_a \psi + \sum_s \pi(s) u'(c(s)) \frac{\partial}{\partial \lambda_a} c(s) = 0\]
\[-q_b \psi + \sum_s \pi(s) u'(c(s)) \frac{\partial}{\partial \lambda_b} c(s) = 0\]

\[-q_a^* \psi + (1 - \tau) \sum_s \pi(s) u'(c(s)) \frac{\partial}{\partial \lambda_a^*} c(s) = 0\]
\[-q_b^* \psi + (1 - \tau) \sum_s \pi(s) u'(c(s)) \frac{\partial}{\partial \lambda_b^*} c(s) = 0\]

\[c(s) = \lambda_a z_a(s) A_a k_a + \lambda_b z_b(s) A_b (1 - k_a) + \lambda_a^* z_a^*(s) A_a^* k_a^* + \lambda_b^* z_b^*(s) A_b^* (1 - k_a^*)\]

(analogous for the foreign consumer)

Three independent equations come out of these FOC’s. Analogously, three more equations are derived from the foreign consumer and 4 more equations represent the stock market clearance conditions:

\[\lambda_a + \eta_a = 1 \quad \lambda_a^* + \eta_a^* = 1\]
\[\lambda_b + \eta_b = 1 \quad \lambda_b^* + \eta_b^* = 1\]

From these 10 equations, one can pin down 8 portfolio choices (for the domestic and foreign consumers) and prices \(p\) and \(p^*\) at which consumers sell their capital to firms.

### 8.2 Definitions

Here, we define the different terms used in the above analysis.

Consumption and Output are defined as:

domestic consumption: \[c(s) = \lambda_a k_a A_a z_a(s) + \lambda_a^* k_a^* A_a^* z_a^*(s) + \lambda_b k_b A_b z_b(s) + \lambda_b^* k_b^* A_b^* z_b^*(s)\]

foreign consumption: \[c^*(s) = \eta_a k_a^* A_a^* z_a^*(s) + \eta_a^* k_a A_a z_a(s) + \eta_b^* k_b^* A_b^* z_b^*(s) + \eta_b k_b A_b z_b(s)\]
domestic output: \( y(s) = k_a A_a z_a(s) + k_b A_b z_b(s) \)

foreign output: \( y^*(s) = k_a^* A_a^* z_a^*(s) + k_b^* A_b^* z_b^*(s) \)

Correlation of domestic consumption and domestic output, one of the measures of consumption risk sharing:

\[
\text{corr}(c, y) = \frac{\text{cov}(c, y)}{\sigma_c \sigma_y}
\]

where,
\[
\text{cov}(c, y) = \lambda_a k_a^2 A_a^2 \sigma^2 + \lambda_a k_a k_b A_a A_b \rho_{ab} \sigma^2 + \\
\lambda_a k_a^* k_a A_a^* A_a \rho_{aa} \sigma^2 + \lambda_a k_a^* k_b A_a^* A_b \rho_{ab} \sigma^2 + \\
\lambda_b k_b A_a A_b \rho_{ab} \sigma^2 + \lambda_b k_b^2 A_b^2 \sigma^2 + \\
\lambda_b k_b^* k_a A_b^* A_a \rho_{ab} \sigma^2 + \lambda_b k_b^* k_b A_b^* A_b \rho_{bb} \sigma^2
\]

\[
\sigma_y^2 = k_a^2 A_a^2 \sigma^2 + 2k_a k_b A_a A_b \rho_{ab} \sigma^2 + k_b^2 A_b^2 \sigma^2
\]

and,
\[
\sigma_c^2 = \lambda_a^2 k_a^2 A_a^2 \sigma^2 + 2\lambda_a \lambda_a^* k_a^* k_a A_a^* A_a \rho_{aa} \sigma^2 + 2 \lambda_a \lambda_b k_b A_a A_b \rho_{ab} \sigma^2 + 2 \lambda_a \lambda_b^* k_b^* k_a A_a^* A_a \rho_{ab} \sigma^2 + 2 \lambda_a \lambda_b^* k_b^* k_b A_a A_b \rho_{ab} \sigma^2 + 2 \lambda_a \lambda_b \lambda_b^* k_b^* k_b A_a A_b \rho_{ab} \sigma^2 + \\
(\lambda_a^*)^2 k_a^2 A_a^2 \sigma^2 + 2 \lambda_a \lambda_b^* k_a^* k_b A_a^* A_b \rho_{ab} \sigma^2 + 2 \lambda_a \lambda_b^* k_a^* k_b^* A_a^* A_b \rho_{ab} \sigma^2 + \\
2 \lambda_a^* \lambda_b^* k_b^* k_a A_a^* A_b \rho_{ab} \sigma^2 + \lambda_b^2 k_b^2 A_b^2 \sigma^2 + 2 \lambda_a \lambda_b \lambda_b^* k_b^* k_b A_a A_b \rho_{bb} \sigma^2 + (\lambda_b^*)^2 k_b^2 A_b^2 \sigma^2
\]

Correlation of domestic and foreign consumption, another measure of consumption smoothing is defined as:

\[
\text{corr}(c, c^*) = \frac{\text{cov}(c, c^*)}{\sigma_c \sigma_{c^*}}
\]
where,

\[ \text{cov}(c, c^*) = E[cc^*] - E[c]E[c^*] \]

\[
= \lambda_a \eta_a k_a k_a^* A_a A_a^* \sigma^2 + \lambda_a \eta_a k_a^2 A_a^2 \sigma^2 + \\
\lambda_a \eta_b k_b k_b^* A_b A_b^* \sigma^2 + \lambda_a \eta_b k_b^2 A_b^2 \sigma^2 + \\
\eta_a^* \lambda_b k_a^* k_b A_b^* A_b^* \rho_{ab} \sigma^2 + \eta_a \lambda_b k_a k_b A_a^* A_b^* \rho_{ab} \sigma^2 + \\
\eta_b^* \lambda_b k_b^2 A_b A_b^* \rho_{ab} \sigma^2 + \lambda_b \eta_b k_b^2 A_b^2 \sigma^2 + \\
\lambda_a \eta_a k_a^2 A_a^* \sigma^2 + \lambda_a \eta_a k_a^2 A_a^* \rho_{aa^*} \sigma^2 + \\
\lambda_a \eta_b k_a^2 A_a^* \sigma^2 + \lambda_a \eta_b k_a^2 A_a^* A_a^* \rho_{aa^*} \sigma^2 + \\
\eta_a^* \lambda_b k_a^2 k_b^2 A_b^* A_b^* \rho_{ab} \sigma^2 + \eta_a \lambda_b k_a^2 k_b^2 A_a^* A_b^* \rho_{ab} \sigma^2 + \\
\eta_b^* \lambda_b k_b^2 A_b^* \rho_{ab} \sigma^2 + \eta_b \lambda_b k_b^2 A_b^* \rho_{bb^*} \sigma^2 
\]

and,

\[ \sigma_{y^*}^2 = \eta_a^2 k_a^2 A_a^2 \sigma^2 + 2 \eta_a k_a k_a^* A_a A_a^* \rho_{aa^*} \sigma^2 + \]

\[ + 2 \eta_a \eta_b k_a k_b A_a^* A_b^* \rho_{ab} \sigma^2 + 2 \eta_b k_b k_b^* A_b A_b^* \rho_{bb^*} \sigma^2 + \\
\eta_a^2 k_a^2 A_a^2 \sigma^2 + 2 \eta_a \eta_b k_a k_b A_a^* A_b^* \rho_{ab} \sigma^2 + \\
2 \eta_a \eta_b k_a k_b A_a A_b \rho_{ab} \sigma^2 + \eta_b^2 k_b^2 A_b^2 \sigma^2 + \\
2 \eta_b k_b k_b^* A_b A_b^* \rho_{bb^*} \sigma^2 + \eta_b^2 k_b^2 A_b^2 \sigma^2 
\]

Output correlation across countries:

\[ \text{corr}(y, y^*) = \frac{\text{cov}(y, y^*)}{\sigma_y \sigma_{y^*}} \]

where,

\[ \text{cov}(y, y^*) = k_a k_a^* A_a A_a^* \sigma^2 + k_B k_B^* A_B A_B^* \sigma^2 + k_b k_b^* A_b A_b^* \rho_{ab} \sigma^2 \]

and,

\[ \sigma_{y^*}^2 = k_a^2 A_a^2 \sigma^2 + 2 k_a k_b A_a A_b \rho_{ab} \sigma^2 + k_b^2 A_b^2 \sigma^2 \]
Figure 1: Portfolio Holdings

a. Cross-Sector Productivity Correlations

b. Cross-Country Productivity Correlations

Note: The x-axis represents impediments to trade in foreign capital. On the right side, frictions in capital markets are high and the markets can be shut (no trade in financial assets takes place). As impediments to foreign capital decrease, we can see the share of domestic assets in home portfolio decrease. Figure 1a shows what happens as productivity in both sectors becomes more correlated and Figure 1b shows what happens when productivity become more correlated across countries.

Figure 2: Industrial Specialization

Figure 3: Output Volatility

Note: The x-axis represents impediments to trade in foreign capital. On the right side, frictions in capital markets are high and the markets can be shut (no trade in financial assets takes place). As impediments to foreign capital decrease, we can see the share of domestic assets in home portfolio decrease. Figure 2 shows what happens to the productive resource allocation in sector A for low and high cross-sector productivity correlations. Figure 3 shows output volatility as impediments to trade in foreign capital decrease.
Figure 4: Industrial Specialization

a. Cross-Country Productivity Correlations (sectors A and B*)

b. Cross-Country Productivity Correlations (sectors A and A*)

Note: The x-axis represents impediments to trade in foreign capital. As impediments to foreign capital decrease, more resources are allocated to sector A (more productive sector). Figure 4a shows what happens to resource allocations when productivity becomes more correlated across sectors and Figure 4b shows resource allocations for each level of financial openness when productivity becomes more correlated across countries.

Figure 5: Output Correlations

Figure 6: Output Volatility

Note: The x-axis represents impediments to trade in foreign capital. As impediments to foreign capital decrease, more resources are allocated to sector A (more productive sector). Figure 5 shows output correlations as countries become more financially open. Figure 6 shows output volatility and financial openness for two different values of cross-country productivity correlations.
Figure 7: Portfolio Choice

Holdings of Domestic Asset "a" at Home

Figure 8: Industrial Specialization

Resource Allocation to Sector "a" at Home

Note: 3-D representations of financial impediments, cross-country productivity correlations and Portfolio Choice (Figure 7) and Industrial Specialization (Figure 8).

Figure 9: Consumption Risk Sharing

a. Consumption Smoothing

b. Cross-country productivity correlations

c. Cross-Sector productivity correlations

d. Productivity Correlations – Horse Race
Figure 10: Consumption Risk Sharing – 3-D

a. Consumption smoothing \( \text{corr}(c, y) \)  
b. Consumption Smoothing \( \text{corr}(c, c^*) \)

Note: The x-axis represents impediments to trade in foreign capital. On the right side, frictions in capital markets are high and the markets can be shut (no trade in financial assets takes place).

Figure 11:  
a. \( \frac{A_a}{A_b} = 1.01 \) (1% more productive)  
b. \( \frac{A_a}{A_b} = 1.02 \) (2% more productive)

Note: The x-axis represents impediments to trade in foreign capital. On the right side, frictions in capital markets are high and the markets can be shut (no trade in financial assets takes place). These graphs show what happens to Expected Consumption as financial openness increases.
Note: The x-axis represents impediments to trade in foreign capital. On the right side, frictions in capital markets are high and the markets can be shut (no trade in financial assets takes place). These graphs show what happens to Portfolio Choice and Industrial Specialization as volatility in Home sector A increases from 0.2 to 0.22.
Figure 16: Consumption smoothing ($\text{corr}(c, y)$)

a. Volatility of Sector $\alpha$ at Home increases

b. Volatility of both Sectors at Home increases

Note: The x-axis represents impediments to trade in foreign capital. On the right side, frictions in capital markets are high and the markets can be shut (no trade in financial assets takes place). These graphs show what happens to Portfolio Choice and Industrial Specialization as volatility in Home in sector A (Figure 16a) and in both sectors (Figure 16b) increases from 0.2 to 0.22.

Figure 17: Portfolio Holdings

Figure 18: Industrial Specialization

Note: The x-axis represents impediments to trade in foreign capital. On the right side, frictions in capital markets are high and the markets can be shut (no trade in financial assets takes place). These Figures represent the case when foreign consumption is an imperfect substitute for domestic consumption.