CHAPTER VII

OF THE COMPETITION OF PRODUCERS

43. Every one has a vague idea of the effects of competition. Theory should have attempted to render this idea more precise; and yet, for lack of regarding the question from the proper point of view, and for want of recourse to symbols (of which the use in this connection becomes indispensable), economic writers have not in the least improved on popular notions in this respect. These notions have remained as ill-defined and ill-applied in their works, as in popular language.

To make the abstract idea of monopoly comprehensible, we imagined one spring and one proprietor. Let us now imagine two proprietors and two springs of which the qualities are identical, and which, on account of their similar positions, supply the same market in competition. In this case the price is necessarily the same for each proprietor. If $p$ is this price, $D = F(p)$ the total sales, $D_1$ the sales from the spring (1) and $D_2$ the sales from the spring (2), then $D_1 + D_2 = D$. If, to begin with, we neglect the cost of production, the respective incomes of the proprietors will be $pD_1$ and $pD_2$; and each of them independently will seek to make this income as large as possible.

We say each independently, and this restriction is very
essential, as will soon appear; for if they should come to
an agreement so as to obtain for each the greatest possible
income, the results would be entirely different, and would
not differ, so far as consumers are concerned, from those
obtained in treating of a monopoly.

Instead of adopting $D = F(p)$ as before, in this case it
will be convenient to adopt the inverse notation $p = f(D)$;
and then the profits of proprietors (1) and (2) will be re-
spectively expressed by

$$D_1 \times f(D_1 + D_2), \text{ and } D_2 \times f(D_1 + D_2),$$

i.e. by functions into each of which enter two variables, $D_1$
and $D_2$.

Proprietor (1) can have no direct influence on the de-
termination of $D_2$: all that he can do, when $D_2$ has been
determined by proprietor (2), is to choose for $D_1$ the value
which is best for him. This he will be able to accomplish
by properly adjusting his price, except as proprietor (2),
who, seeing himself forced to accept this price and this
value of $D_1$, may adopt a new value for $D_3$, more favourable
to his interests than the preceding one.

Analytically this is equivalent to saying that $D_1$ will be
determined in terms of $D_3$ by the condition

$$\frac{d[D_3 f(D_1 + D_2)]}{dD_1} = 0,$$

and that $D_2$ will be determined in terms of $D_1$ by the
analogous condition

$$\frac{d[D_2 f(D_1 + D_2)]}{dD_2} = 0,$$
OF THE THEORY OF WEALTH

whence it follows that the final values of \( D_1 \) and \( D_2 \), and consequently of \( D \) and of \( P \), will be determined by the system of equations

\[
\begin{align*}
(1) & \quad f(D_1 + D_2) + D_1 f'(D_1 + D_2) = 0, \\
(2) & \quad f(D_1 + D_2) + D_2 f'(D_1 + D_2) = 0.
\end{align*}
\]

Let us suppose the curve \( m_1 n_1 \) (Fig. 2) to be the plot of equation (1), and the curve \( m_2 n_2 \) that of equation (2), the variables \( D_1 \) and \( D_2 \) being represented by rectangular coordinates. If proprietor (1) should adopt for \( D_1 \) a value represented by \( ox_1 \), proprietor (2) would adopt for \( D_2 \) the value \( oy_1 \), which, for the supposed value of \( D_2 \), would give him the greatest profit. But then, for the same reason, producer (1) ought to adopt for \( D_1 \) the value \( ox_1h \), which gives the maximum profit when \( D_2 \) has the value \( oy_1 \). This would bring producer (2) to the value \( oy_1h \) for \( D_2 \), and so forth; from which it is evident that an equilibrium can only be established where the coordinates \( ox \) and \( oy \) of the point of intersection \( i \) represent the values of \( D_1 \) and \( D_2 \). The same construction repeated on a point of the figure on the other side of the point \( i \) leads to symmetrical results.

The state of equilibrium corresponding to the system of values \( ox \) and \( oy \) is therefore stable; i.e., if either of the producers, misled as to his true interest, leaves it temporarily, he will be brought back to it by a series of reactions, constantly declining in amplitude, and of which the dotted lines of the figure give a representation by their arrangement in steps.

The preceding construction assumes that \( om_1 > om_2 \) and \( on_1 < on_2 \); the results would be diametrically opposite if
THE MATHEMATICAL PRINCIPLES

these inequalities should change sign, and if the curves intersect, would then cease to correspond to a state of stable equilibrium. But it is easy to prove that such disposition of the curves is inadmissible. In fact, if \( D_1 = 0 \), equations (1) and (2) reduce, the first to

\[
f(D_1) = 0,
\]

and the second to

\[
f(D_2) + D_2 f'(D_2) = 0.
\]

The value of \( D_2 \) derived from the first would correspond to \( \rho = 0 \); the value of \( D_2 \) derived from the second corresponds to a value of \( \rho \) which would make the product \( \rho D_2 \) a maximum. Therefore the first root is necessarily greater than the second, or \( \omega_1 > \omega_m \), and for the same reason

\[ o_m > \omega_m. \]

We can derive first \( D_1 = D_2 \) (which ought to be the case, as the springs are supposed to be similar and similarly situated), and then by addition:

\[
2 f(D) + D f'(D) = 0,
\]

an equation which can be transformed into

\[
D + 2 \rho \frac{dD}{d\rho} = 0,
\]

whereas, if the two springs had belonged to the same property, or if the two proprietors had come to an understanding, the value of \( \rho \) would have been determined by the equation

\[
D + \rho \frac{dD}{d\rho} = 0.
\]
OF THE THEORY OF WEALTH

and would have rendered the total income $Dp$ a maximum, and consequently would have assigned to each of the producers a greater income than what they can obtain with the value of $\rho$ derived from equation (3).

Why is it then that, for want of an understanding, the producers do not stop, as in the case of a monopoly or of an association, at the value of $\rho$ derived from equation (4), which would really give them the greatest income?

The reason is that, producer (1) having fixed his production at what it should be according to equation (4) and the condition $D_1 = D_2$, the other will be able to fix his own production at a higher or lower rate with a temporary benefit. To be sure, he will soon be punished for his mistake, because he will force the first producer to adopt a new scale of production which will react unfavourably on producer (2) himself. But these successive reactions, far from bringing both producers nearer to the original condition [of monopoly], will separate them further and further from it. In other words, this condition is not one of stable equilibrium; and, although the most favourable for both producers, it can only be maintained by means of a formal engagement; for in the moral sphere men cannot be supposed to be free from error and lack of forethought any more than in the physical world bodies can be considered perfectly rigid, or supports perfectly solid, etc.

45. The root of equation (3) is graphically determined by the intersection of the line $y = 2x$ with the curve $y = \frac{F(x)}{F'(x)}$; while that of equation (4) is graphically shown by the intersection of the same curve with the line $y = x$. 
But, if it is possible to assign a real and positive value to
the function \( y = -\frac{F(x)}{F'(x)} \) for every real and positive value of
\( x \), then the abscissa \( x \) of the first point of intersection will
be smaller than that of the second, as is sufficiently proved
simply by the plot of Fig. 4. It is easily proved also that
the condition for this result is always realized by the very
nature of the law of demand. In consequence the root
of equation (3) is always smaller than that of equation (4) ;
or (as every one believes without any analysis) the result of
competition is to reduce prices.

46. If there were 3, 4, \ldots, \( n \) producers in competition,
all their conditions being the same, equation (3) would be
successively replaced by the following:

\[
D + 3\dot{p}\frac{dD}{dp} = o, \quad D + 4\dot{p}\frac{dD}{dp} = o, \quad \ldots \quad D + n\dot{p}\frac{dD}{dp} = o;
\]

and the value of \( \dot{p} \) which results would diminish indefinitely
with the indefinite increase of the number \( n \).

In all the preceding, the supposition has been that natural
limitation of their productive powers has not prevented pro-
ducers from choosing each the most advantageous rate of
production. Let us now admit, besides the \( n \) producers,
who are in this condition, that there are others who reach
the limit of their productive capacity, and that the total
production of this class is \( \Delta \); we shall continue to have
the \( n \) equations

\[
\begin{cases}
f(D) + D_1f'(D) = o, \\
f(D) + D_2f'(D) = o, \\
\quad \cdots \quad \cdots \quad \cdots \quad \cdots \\
f(D) + D_nf'(D) = o,
\end{cases}
\]
which will give \( D_1 = D_2 = \cdots = D_n \) and by addition,
\[
nf(D) + nD_n f'(D) = 0.
\]
But \( D = nD_1 + \Delta \), whence
\[
nf(D) + (D - \Delta)f'(D) = 0,
\]
or
\[
D - \Delta + np\frac{dD}{dp} = 0.
\]

This last equation will now replace equation (3) and determine the value of \( \rho \) and consequently of \( D \).

47. Each producer being subject to a cost of production expressed by the functions \( \phi_1(D_1), \phi_2(D_2), \cdots, \phi_n(D_n) \), the equations of (5) will become
\[
\begin{align*}
f(D) + D_1 f'(D) - \phi_1'(D_1) &= 0, \\
f(D) + D_2 f'(D) - \phi_2'(D_2) &= 0, \\
&\vdots \\
f(D) + D_n f'(D) - \phi_n'(D_n) &= 0.
\end{align*}
\]

If any two of these equations are combined by subtraction, for instance if the second is subtracted from the first, we shall obtain
\[
D_1 - D_2 = \frac{1}{f'(D)} \left[ \phi_1'(D_1) - \phi_1'(D_2) \right]
= \frac{dD}{dp} \left[ \phi_1'(D_1) - \phi_1'(D_2) \right].
\]

As \( \frac{dD}{dp} \) is essentially negative, we shall therefore have at the same time
\[
D_1 \gtrless D_2 \quad \text{and} \quad \phi_1'(D_1) \gtrless \phi_1'(D_2).
\]
Thus the production of plant $A$ will be greater than that of plant $B$, whenever it will require greater expense to increase the production of $B$ than to increase the production of $A$ by the same amount.

For a concrete example, let us imagine the case of a number of coal mines supplying the same market in competition one with another, and that, in a state of stable equilibrium, mine $A$ markets annually 20,000 hectariters and mine $B$, 15,000. We can be sure that a greater addition to the cost would be necessary to produce and bring to market from mine $B$ an additional 1000 hectariters than to produce the same increase of 1000 hectariters in the yield of mine $A$.

This does not make it impossible that the costs at mine $A$ should exceed those at mine $B$ at a lower limit of production. For instance, if the production of each were reduced to 10,000 hectariters, the costs of production at $B$ might be smaller than at $A$.

48. By addition of equations (6), we obtain

$$nf(D) + Df'(D) - \sum \phi_i'(D_n) = 0,$$

or

$$D + \frac{dD}{dp} [np - \sum \phi_i'(D_n)] = 0.$$

If we compare this equation with the one which would determine the value of $p$ in case all the plants were dependent on a monopolist, viz.

$$D + \frac{dD}{dp} [p - \phi'(D)] = 0,$$

we shall recognize that on the one hand substitution of the
term \( np \) for the term \( \rho \) tends to diminish the value of \( \rho \); but on the other hand substitution of the term \( \mathbb{Z} \phi_n'(D_n) \) for the term \( \phi'(D) \) tends to increase it, for the reason that we shall always have
\[
\mathbb{Z} \phi_n'(D_n) > \phi'(D);
\]
and, in fact, not only is the sum of the terms \( \phi_n'(D_n) \) greater than \( \phi'(D) \), but even the average of these terms is greater than \( \phi'(D) \), i.e. we shall have the inequality
\[
\frac{\mathbb{Z} \phi_n'(D_n)}{n} > \phi'(D).
\]

To satisfy one's self of this, it is only necessary to consider that any capitalist, holding a monopoly of productive property, would operate by preference the plants of which the operation is the least costly, leaving the others idle if necessary; while the least favoured competitor will not make up his mind to close his works so long as he can obtain any profit from them, however modest. Consequently, for a given value of \( \rho \), or for the same total production, the costs will always be greater for competing producers than they would be under a monopoly.

It now remains to be proved that the value of \( \rho \) derived from equation (8) is always greater than the value of \( \rho \) derived from equation (7).

For this we can see at once that if in the expression \( \phi'(D) \) we substitute the value \( D = F(\rho) \), we can change \( \phi'(D) \) into a function \( \psi(\rho) \); and each of the terms which enter into the summational expression \( \mathbb{Z} \phi_n'(D_n) \), can also be regarded as an implicit function of \( \rho \), in virtue of the
relation $D = F(p)$ and of the system of equations (6). In consequence the root of equation (7) will be the abscissa of the point of intersection of the curve

$$(a) \quad y = \frac{F(x)}{F'(x)}$$

with the curve

$$(b) \quad y = nx - [\psi_1(x) + \psi_2(x) + \cdots + \psi_n(x)] ;$$

while the root of equation (8) will be the abscissa of the point of intersection of the curve $(a)$ with one which has for its equation

$$(b') \quad y = x - \psi(x).$$

As has been already noted, equation $(a)$ is represented by the curve $MN$ (Fig. 5), of which the ordinates are always real and positive; we can represent equation $(b)$ by the curve $PQ$, and equation $(b')$ by the curve $P'Q'$. In consequence of the relation just proved, viz.,

$$\sum \psi_n(x) > \psi(x),$$

we find for the value $x = 0$, $OP > OP'$. It remains to be proved that the curve $P'Q'$ cuts the curve $PQ$ at a point $J$ situated below $MN$, so that the abscissa of the point $Q'$ will be greater than that of the point $Q$.

This amounts to proving that at the points $Q$ and $Q'$, the ordinate of the curve $(b)$ is greater than the ordinate of the curve $(b')$ corresponding to the same abscissa.

Suppose that it were not so, and that we should have

$$x - \psi(x) > nx - [\psi_1(x) + \psi_2(x) + \cdots + \psi_n(x)],$$

or

$$(n - 1)x < \psi_1(x) + \psi_2(x) + \cdots + \psi_n(x) - \psi(x).$$
\( \psi(x) \) is an intermediate quantity between the greatest and smallest of the terms \( \psi_1(x), \psi_2(x), \ldots, \psi_{n-1}(x), \psi_n(x) \); if we suppose that \( \psi_n(x) \) denotes the smallest term of this series, the preceding inequality will involve the following inequality:

\[(n - 1) x < \psi_1(x) + \psi_2(x) + \cdots + \psi_{n-1}(x).\]

Therefore \( x \) will be smaller than the average of the \( n - 1 \) terms of which the sum forms the second member of the inequality; and among these terms there will be some which are greater than \( x \). But this is impossible, because producer \( (k) \), for instance, will stop producing as soon as \( \rho \) becomes less than \( \phi'(D_1) \) or \( \psi_1(\rho) \).

49. Therefore if it should happen that the value of \( \rho \) derived from equations (6), combined with the relations

\[(g) \quad D_1 + D_2 + \cdots + D_n = D, \quad \text{and} \quad D = F(\rho),\]

should involve the inequality

\[\rho - \phi'(D_1) < 0,\]

it would be necessary to remove the equation

\[f'(D) + D_s f'(D) - \phi'(D_1) = 0\]

from the list of equations (6), and to substitute for it

\[\rho - \phi'(D_2) = 0,\]

which would determine \( D_2 \) as a function of \( \rho \). The remaining equations of (6), combined with equations (g), will determine all the other unknown quantities of the problem.