Replaces Example 7.3
Pages 283-285, Jehle & Reny, 2nd. Ed.

EXAMPLE 7.3 To see these ideas at work, let's consider in more detail the two firms discussed in Example 7.2. Suppose that firm 1's marginal cost of production is zero. Also, suppose that firm 2's marginal cost is either 1 or 4, and that each of these "types" of firm 2 occur with probability 1/2. If the lowest price charged is p, then market demand is $8 - p$. To keep things simple, we'll suppose that each firm can choose only one of three prices, 1, 4, or 6. The payoffs to the firms are described in Fig. 7.7. Firm 1's payoff is always the first number in any pair, and firm 2's payoff when his costs are low (high) are given by the second number in the entries of the matrix on the left (right).

In keeping with the Bertrand-competition nature of the problem, we have instituted the following convention in determining payoffs when the firms choose the same price. If both firms' costs are strictly less than the common price, then the market is split evenly between them. Otherwise, firm 1 captures the entire market at the common price. The latter uneven split reflects the idea that if the common price is above only firm 1's cost, firm 1 could capture the entire market by lowering his price slightly (which, if we let him, he could do and still more than cover his costs), whereas firm 2 would not lower his price (even if we let him) because this would result in losses.

We have now described the game of incomplete information. The associated strategic form game is one in which there are three players: firm 1, firm $2l$ (low cost), and firm $2h$ (high cost). Each has the same pure strategy set, namely, the set of prices $\{1, 4, 6\}$. Let p_1, p_l, p_h denote the price chosen by firms 1, $2l$, and $2h$, respectively.

$1 \setminus 2l$	$p_l = 6$	$p_l = 4$	$p_l = 1$
$p_1 = 6$	6, 5	0, 12	0, 0
$p_1 = 4$	16, 0	8, 6	0, 0
$p_1 = 1$	7, 0	7, 0	7, 0

$1 \setminus 2h$	$p_h = 6$	$p_h = 4$	$p_h = 1$
$p_1 = 6$	6, 2	0, 0	0, -21
$p_1 = 4$	16, 0	16, 0	0, -21
$p_1 = 1$	7, 0	7, 0	7, 0

Figure 7.7. A Bertrand-competition incomplete information game.

Firm 1 chooses $p_1 = 6$

$2l \setminus 2h$	$p_h = 6$	$p_h = 4$	$p_h = 1$
$p_l = 6$	6, 5, 2	3, 5, 0	3, 5, -21
$p_l = 4$	3, 12, 2	0, 12, 0	0, 12, -21
$p_l = 1$	3, 0, 2	0, 0, 0	0, 0, -21

Firm 1 chooses $p_1 = 4$

$2l \setminus 2h$	$p_h = 6$	$p_h = 4$	$p_h = 1$
$p_l = 6$	16, 0, 0	16, 0, 0	8, 0, -21
$p_l = 4$	12, 6, 0	12, 6, 0	4, 6, -21
$p_l = 1$	8, 0, 0	8, 0, 0	0, 0, -21

Firm 1 chooses $p_1 = 1$

$2l \setminus 2h$	$p_h = 6$	$p_h = 4$	$p_h = 1$
$p_l = 6$	7, 0, 0	7, 0, 0	7, 0, 0
$p_l = 4$	7, 0, 0	7, 0, 0	7, 0, 0
$p_l = 1$	7, 0, 0	7, 0, 0	7, 0, 0

Figure 7.8. The associated strategic form game.

Fig. 7.8 depicts this strategic form game. As there are three players, firm 1's choice of price determines the matrix, and firms $2l$ and $2h$'s prices determine the row and column, respectively, of the chosen matrix. For example, according to Fig. 7.8, if firm 1 chooses $p_1 = 4$, firm $2l$ $p_l = 4$, and firm $2h$ $p_h = 4$, their payoffs would be 12, 6, and 0, respectively.

According to Definition 7.11, the payoffs in the strategic form game of Fig. 7.8 for firms $2l$ and $2h$ can be obtained by simply reading them off of the matrices from Fig. 7.7. This is because there is only one "type" of firm 1. For example, according to Fig. 7.7, if firm $2l$ is chosen by Nature and chooses $p_l = 6$, then it receives a payoff of 5 if firm 1 chooses $p_1 = 6$. Note that this is reflected in the associated game of Fig. 7.8, where firm $2l$'s payoff is 5 when it and firm 1 choose a price of 6 regardless of the price chosen by firm $2h$.

The payoffs to firm 1 in the associated strategic form game of Fig. 7.8 are obtained through the use of the prior distribution on firm 2's costs. For example, consider the strategy in which firm $2l$ chooses $p_l = 1$, firm $2h$ chooses $p_h = 6$, and firm 1 chooses $p_1 = 4$. Now, if firm $2l$ is chosen (by Nature), then according to Fig. 7.7, firm 1's payoff is zero. If firm $2h$ is chosen, then firm 1's payoff is 16. Because each type of firm 2 is chosen with probability 1/2 according to the prior, firm 1's expected payoff is 8. This is precisely firm 1's payoff corresponding to $p_1 = 4, p_l = 1$, and $p_h = 6$ in Fig. 7.8. One can similarly calculate firm 1's associated strategic form game (expected) payoff given in Fig. 7.8 for all other joint strategy combinations.

To discover a Bayesian-Nash equilibrium of the Bertrand-competition incomplete information game, we must look for a Nash equilibrium of the associated strategic form game of Fig. 7.8.

Finding one Nash equilibrium is particularly easy here. Note that firms $2l$ and $2h$ each have a weakly dominant strategy: choosing a price of 4 is weakly dominant for firm $2l$ and choosing a price of 6 is weakly dominant for firm $2h$. But once we eliminate the other strategies for them, firm 1 then has a strictly dominant strategy, namely, to choose a price of 4. To see this, suppose that $p_l = 4$ and $p_h = 6$. Then according to Fig. 7.8, firm 1's payoff is 3 if he chooses $p_1 = 6$, 12 if he chooses $p_1 = 4$, and 7 if he chooses $p_1 = 1$.

Consequently, there is a pure strategy Bayesian-Nash equilibrium in which two of the three firms choose a price of 4 while the third chooses a price of 6. You are invited to explore the existence of other Bayesian-Nash equilibria of this game in an exercise. Note that in contrast to the case of Bertrand competition with complete information, profits are not driven to zero here. Indeed, only the high-cost firm 2 earns zero profits in the equilibrium described here. □