

# Is it Really the Fisher Effect?

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Many researchers have used a cointegration approach to test for the Fisher effect. This note argues that the cointegration of the nominal interest rate and the inflation rate is consistent with any theory implying a stationary *ex post* real interest rate and so is not a sufficient condition for the Fisher effect to hold. The sufficient condition is the unpredictability of the inflation forecast error implied by the nominal interest rate and this condition may be tested using the signal extraction framework of Durlauf and Hall (1988, 1989).

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A series of articles in this and its companion journals, including Atkins (1989), MacDonald and Murphy (1989), Bonham (1991), Moazzami (1991), Dutt and Ghosh (1995), Daniels, Nourzad and Toutkoushian (1996), Lee, Clark and Ahn (1998), Carneiro, Divino and Rocha (2002), and, Granville and Mallick (2004), have attempted to examine the role of the Fisher effect in determining nominal interest rates by testing for cointegration between nominal interest rates and inflation rates.<sup>1</sup> In this note I explain why the finding of cointegration between nominal interest rates and inflation rates is largely uninformative about the existence of the Fisher effect. I then describe an alternative approach that examines all testable implications of Fisher's theory and report the results of applications of that approach.

Let  $i_t$  be the time  $t$  one-period-ahead nominal interest rate and  $\pi_{t+1}$  be the inflation rate between periods  $t$  and  $t + 1$ . Assume that both  $i_t$  and  $\pi_{t+1}$  obey stochastic processes that are integrated of order one. An obvious candidate cointegrating vector is  $(1, -1)$  as this linear combination of  $i_t$  and  $\pi_{t+1}$  is the *ex post* real interest rate  $r_t = i_t - \pi_{t+1}$ . Growth theory, however, shows that long-run real interest rates are determined in the steady state of the economy and so the stationarity of  $r_t$  has no implications for the veracity of the Fisher effect. That stationarity is consistent with a host of theories of nominal interest rate behavior and therefore can, at most, be a necessary condition for the Fisher effect to hold.<sup>2</sup> Moreover, there can be just one cointegrating vector here, so the existence of any other stationary linear combination of  $i_t$  and  $\pi_{t+1}$  implies that  $r_t$  is not stationary. This possibility is inconsistent with long-run real interest rates being determined in the steady state of the economy as well as with Cochrane (1991)'s argument that interest rates are “almost certainly” not integrated because, if they are, the observed similarity in the values of interest rates now and in the distant past is an extremely low probability event.<sup>3</sup> Thus, if inflation and nominal interest rates are integrated, the Fisher effect implies, but

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<sup>1</sup>Examples from other journals include Mishkin (1992), Wallace and Warner (1993), Mishkin and Simon (1995), Peláez (1995), Crowder and Hoffman (1996), Crowder (1997), Koustas and Serletis (1999), and, Fahmy and Kandil (2003).

<sup>2</sup>Miron (1991) makes the same point in the context of testing the expectations theory of the term structure.

<sup>3</sup>Cochrane's view alone renders the results of cointegration tests of the Fisher effect meaningless as, absent integration, the concept of cointegration is vacuous. I confess that Johnson (1994b) also tests the integration and cointegration properties of interest rate data but do not further discuss the issue here.

is not implied by, their cointegration. Of course, if inflation and nominal interest rates are not integrated, no linear combination of  $i_t$  and  $\pi_{t+1}$ , including the real interest rate, is integrated. In either case, a finding of a stationary real interest rate is largely uninformative.

The sufficient condition for the Fisher effect to hold is that nominal interest rates embody an optimal inflation forecast – a condition that can be tested using the signal extraction approach for testing expectations-based models described in Durlauf and Hall (1988, 1989). To apply the Durlauf and Hall approach, note that the time  $t$  one-period-ahead nominal interest rate predicted by the rational expectations variant of Fisher's theory of interest,  $i_t^*$ , is given by  $i_t^* \equiv \rho_t + E_t\pi_{t+1}$  where  $\rho_t$  is the one-period-ahead *ex ante* real interest rate at time  $t$  and  $E_t\pi_{t+1}$  is the expectation of  $\pi_{t+1}$  conditional on information available at time  $t$ ,  $\Phi_t$ . The difference between  $i_t^*$  and the observed nominal interest rate,  $i_t$ , is the specification error in Fisher's theory given by  $N_t \equiv i_t - i_t^*$ . Durlauf and Hall refer to this error as “model noise” and show that all testable implications of Fisher's theory can be expressed as the hypothesis that  $N_t = 0$ .<sup>4</sup> Using the definition of  $i_t^*$ , the model noise can be written as  $N_t = i_t - \rho_t - E_t\pi_{t+1} = i_t - \rho_t - \pi_{t+1} + \epsilon_{t+1}$ , where  $\epsilon_{t+1} \equiv \pi_{t+1} - E_t\pi_{t+1}$  is the inflation forecast error satisfying  $E_t\epsilon_{t+1} = 0$  by construction. The *ex post* real interest rate,  $r_t = i_t - \pi_{t+1}$ , can then be written as  $r_t = \rho_t + N_t - \epsilon_{t+1}$ . Under the maintained hypothesis that  $\rho_t$  is constant, and assuming that the time-series properties of  $r_t$  and  $X_t$  are such that the projection is well defined, Fisher's theory can be tested by projecting  $r_t$  onto any  $X_t \subset \Phi_t$  containing a constant. Conditional on the choice of  $X_t$ , the variance of this projection can be shown to be the tightest possible lower bound on that of  $N_t$ .<sup>5</sup> The ratio of the variance of the projection to the variance of  $r_t$  is the  $R^2$  from the regression of  $r_t$  on  $X_t$  and a test of Fisher's theory is thus given by the usual test of the hypothesis that  $R^2 = 0$  in that regression. Moreover, the economic importance of any rejections may be gauged by the magnitude of  $R^2$  as

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<sup>4</sup>As  $i_t^*$  is the risk-free rate in the version of Fisher's theory discussed here so any role played by risk aversion in determining  $i_t$  will be reflected in the model noise.

<sup>5</sup>Durlauf and Hall (1988, 1989) show that all other linear tests are special cases of this approach. Garcia (1993) and Johnson (1994a) demonstrate this proposition for some of the tests of Fisher's theory in the literature by reinterpreting them in the signal extraction framework.

a small, but statistically significant value of  $R^2$ , while implying rejection of the theory, suggests that it is a close approximation to reality.

Garcia (1993) applies this approach to Brazilian data for the period 1973–1990 and, finding little model noise, concludes that the Fisher effect describes the data reasonably well. Johnson (1994a) applies the approach to monthly US data and finds that, while the Fisher's theory can be formally rejected, over the period 1953:01 to 1979:10 it provides a reasonably good description of interest rate behavior. The quality of this description after 1979:10 appears much worse but the small sample sizes prevent definitive conclusions. Johnson and Garcia (2000) relax the assumption of a constant *ex ante* real interest rate and test Fisher's theory after removing an estimate of  $\rho_t$  from the data.<sup>6</sup> They conclude that the 90-day US T-Bill rate over the period 1951:Q4 to 1991:Q4 “... can be well described as the sum of a rational forecast of inflation and an infrequently changing *ex ante* real interest rate” (p. 176).

By writing the *ex post* real interest rate as  $r_t = \rho_t + N_t - \epsilon_{t+1}$  and maintaining the hypothesis of a constant *ex ante* real interest rate,  $\rho_t = \rho$ , the result that all testable implications of Fisher's theory of interest are contained in the proposition that  $N_t = 0$  shows that the strong testable implication of Fisher's theory is that  $r_t$  be orthogonal to  $\Phi_t$ . This requires that  $r_t$  be stationary so if  $i_t$  and  $\pi_{t+1}$  are integrated they must be cointegrated. However, the issue of whether or not  $i_t$  and  $\pi_{t+1}$  can be integrated notwithstanding, the stationarity of  $r_t$  does not imply that  $r_t$  is orthogonal to  $\Phi_t$  and is thus consistent with many models. The stationarity of  $r_t$  is thus only a necessary condition for the existence of the Fisher effect. Put another way, the cointegration of  $i_t$  and  $\pi_{t+1}$  implies only that the variance of the specification error is finite but the veracity of Fisher's theory requires that it be zero. The finding that interest rates and inflation rates are cointegrated is thus, at best, only mildly informative about the usefulness of Fisher's theory.

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<sup>6</sup>If  $\rho_t$  were observable, Fisher's theory could be tested by projecting  $r_t - \rho_t = N_t - \epsilon_{t+1}$  onto any  $X_t \in \Phi_t$ .

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