When Do First-Movers Have an Advantage?

A Stackelberg Classroom Experiment

by

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Abstract

The timing of moves can dramatically affect firm profits and market outcomes. When firms choose output quantities, there is a first-mover advantage and when firms choose prices, there is a second-mover advantage. Students often find it difficult to understand the differences between these two situations. This classroom experiment simulates each scenario in a way that makes it easy for students to understand the theoretical reasons for the different possible outcomes. We develop a two-firm classroom experiment where students first play a Stackelberg game in which firms sequentially choose production quantities and then a Stackelberg game in which firms sequentially choose prices. When choosing quantities, it is advantageous to move first and when choosing prices, it is advantageous to wait.

JEL Codes: A22, C72, L13

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Students often respond enthusiastically when introduced to the analysis of strategic firm interaction. They frequently observe such interaction in real markets and recognize that a deeper understanding of it could be useful to them. However, formal mathematical analysis of firm behavior can be intimidating for many students. Introducing students to strategic thinking through a classroom experiment can capitalize on their initial interest, build intuition for some fundamental principles of strategic behavior, and better prepare them for a more formal treatment of the subject to follow.¹

This paper describes a classroom experiment designed to simulate the impact of sequential decision-making by competing firms. We study the strategic interaction between two firms that produce differentiated substitute products, like soft drinks produced by Pepsi and Coke, or breakfast cereals produced by Kellogg's and Quaker. Using this experiment, we find that students develop a deeper understanding of how firms interact when making sequential decisions and of the situations in which being a leader, or a follower, allows a firm to obtain larger profits.

It is common to use simultaneous-move models (both Cournot and Bertrand) to study the effects of market structure, product differentiation, production efficiency, research and development investments, and many other economic influences on prices and firm profits and efficiency. These models are often starting points for the study of strategic firm interaction.² After students study a simultaneous-move framework, they often move on to Stackelberg games in which one firm acts before another firm. In Stackelberg games, being able to act first is an advantage in certain situations and a disadvantage in others. Understanding the conditions that lead to there being a first-mover advantage (or disadvantage) can be important when a firm is choosing
between charging ahead or waiting to see what a competitor will do. While students appreciate the applicability of Stackelberg games, we find that they often struggle to fully understand the intuition for the different results.

In a Stackelberg game the firms make decisions sequentially rather than simultaneously. The second mover observes the choice of the first mover and chooses its best action accordingly. This means that the choice made by the first mover affects the choice of the second mover. The first mover wants to understand the possible reactions of the second mover in order to make the best choice it can. In general, it is not clear whether being a first mover is an advantage or a disadvantage; the advantage differs from a Stackelberg Cournot (quantity choice) game to a Stackelberg Bertrand (price choice) game. The exercise we describe here makes it easy for students to gain an intuitive understanding of the effect of sequential choices by firms.

The main difference between a Cournot game and a Bertrand game is the choice variable of the firms: in a Cournot game, firms choose the quantity of their good to produce; in a Bertrand game, firms choose the price to charge for their good. This difference in choice variable affects whether the first mover or second mover has a potential advantage in terms of the amount of profit they can earn.

This experiment is most appropriate for courses in industrial organization, game theory, intermediate microeconomics and even international trade. It could also be used in business courses that study pricing strategy and the impact of firm leadership. It requires only the handouts included in the appendix and a coin (or other binary randomizing mechanism) and can
be run in a single class period. (Including time for setup at the beginning and debriefing at the end, the experiment can be run in less than 45 minutes.) The experiment works best in smaller classes of 25 or less, and we include suggestions to keep students in large courses engaged if they are not directly participating in the experiment.

**RELEVANT LITERATURE**

The literature contains a plethora of experiments appropriate for introductory courses but fewer experiments (albeit an ever-increasing number) designed especially for intermediate and advanced field-specific courses. Our review of the literature revealed only a handful of papers describing classroom experiments on Stackelberg and Bertrand interactions. In an experiment structured somewhat similarly to ours, Nelson and Beil (1995) demonstrates the principles of decision-making in dynamic oligopolies, especially the difficulties in forming and maintaining cartels. As an illustration of firm behavior under imperfect competition, their game distinguishes between procedurally rational choices and substantively rational decisions in the context of collusive, Cournot, and competitive equilibria. However, the focus of their exercise is on the potential benefits of collusion rather than the competitive issues that arise in oligopolies. Their exercise also differs from ours in that they use a simultaneous choice framework rather than a sequential choice framework.

Other papers present exercises that focus on a narrow aspect of firm dynamics. For example, Ortmann (2003) describes a one-shot Bertrand pricing experiment to demonstrate the incentive of Bertrand competitors to attempt to undercut the price of other producers. He describes it as a
“demonstration” rather than an experiment because of its brevity. His goal appears to be to demonstrate the single issue of price undercutting rather than to involve the students more fully. More recently, Waldron, Allgrunn, and Pei (2010) describes a Bertrand experiment in which students face different production costs in different rounds. The main goal is to show that Bertrand competition pushes prices down to marginal costs. Winchester (2006) presents a classroom exercise in which students act as different countries and choose tariffs. The focus is on the welfare effects countries may experience from tariffs, so it is somewhat similar to our emphasis on firm profits. However, by focusing on the effect of sequentiality in a market with product differentiation, we generate more-general results. Finally, Beckman (2003) presents an experiment that is similar in structure to ours, but reveals all profit information to all participants, which allows for collusion and other, non-competitive strategies. Also, his exercise requires more than one class period.

We take the fact that these papers have been published as evidence that instructors desire more non-lecture pedagogical options for the topic of firm dynamics. Our goal is to provide a more general model and framework that instructors can modify, if desired, for a diversity of applications, or they can retain the general nature of our experiment to demonstrate robust theoretical results. In particular, our experiment differs from others in the following two key ways:

1. We focus on differences in timing; students come to understand that the optimal timing choice depends on the nature of the choice variable.

2. Our setup allows for different degrees of product differentiation. The instructor can choose the magnitude of product differentiation and can discuss how product
differentiation affects the prices, profits, etc. Allowing product differentiation is especially important in a Bertrand game where, without product differentiation, a firm can increase profit by following a very simple strategy: set their price slightly below the competitor's price.

THEORETICAL MODEL

There are two firms, each producing a unique good. Consumers consider the goods to be imperfect substitutes, meaning that the market price of a good is affected not only by the quantity available of that good, but also by the quantity available of the other good. This, in turn, implies that the profits of each firm are dependent on the choices of both firms. The assumption that the goods are differentiated also implies that the impact of production of the goods on the price of each good does not have to be symmetrical.

Following Singh and Vives (1984), we assume the two firms face linear demands given by:

\[
p_1 = \alpha - \beta q_1 - \gamma q_2 \quad (1)
\]

\[
p_2 = \alpha - \beta q_2 - \gamma q_1 \quad (2)
\]

where \(q_i\) and \(p_i\) respectively denote the quantity and price for the product produced by Firm \(i\), \(i = 1,2\). Both \(\beta\) and \(\gamma\) are assumed to be positive, implying the goods are substitutes. For simplicity, we assume that \(\gamma < \beta\), meaning that the price of a good is more sensitive to changes in the quantity produced of that good than it is to changes in the quantity produced of the other good.\(^3\)

The degree of product differentiation is measured by the fraction \(\frac{\gamma}{\beta} \in (0,1)\). Note that, for a given \(\beta\), product differentiation decreases the closer \(\gamma\) is to \(\beta\), and increases as \(\gamma\) approaches zero.
For simplicity, we focus on firms with identical constant returns to scale production functions, which allows us to abstract from effects of different production technologies and enables us to focus on the effects of the timing of the strategic interaction between the two firms. We assume the marginal cost of production is \( c \geq 0 \) for both firms. We further assume that \( \alpha > c \), which is necessary for both firms to make positive profits in equilibrium for both Cournot and Bertrand games.

**Stackelberg Cournot Model**

In a Stackelberg Cournot game, one firm (called Firm 1) moves first and chooses a quantity to produce. The second firm (Firm 2) observes this quantity choice and responds to it by choosing its own quantity, \( q_2 \), to maximize its profits. If both firms know the market demand and each other's profit maximization problems, Firm 1 can predict the best responses of Firm 2 and will take them into account when choosing \( q_1 \). However, Firm 2 cannot affect Firm 1's choices and has to take \( q_1 \) as a given. Working backwards, we use demand equation (2) to write Firm 2's profit maximization problem as

\[
\max_{q_2} (\alpha - \beta q_2 - \gamma q_1 - c) q_2.
\]

The solution to this problem is the best response function for Firm 2 as a function of the quantity chosen by Firm 1:

\[
q_2(q_1) = \frac{\alpha - \gamma q_1 - c}{2\beta}.
\]  

(3)

Note that Firm 2's best response function is a downward sloping function of \( q_1 \). In general, a firm's best response function will be downward sloping when the firms are choosing an output
quantity. This implies that when one firm chooses a relatively large quantity, the other firm will want to choose a relatively small quantity and vice versa. In this situation, the firm that gets to choose first generally has an advantage in terms of the amount of profit they can make.

Use the demand equation (1) to write the profit maximization problem of Firm 1 as

$$\text{max}_{q_1} (\alpha - \beta q_1 - \gamma q_2(q_1) - c)q_1$$  \hspace{1cm} (4)

where $q_2(q_1)$ (from equation (3)) gives the best response for Firm 2 as a function of $q_1$.

For this experiment, we assume parameter values $\alpha = 342$, $\beta = 10$, and $\gamma = 8$. We further assume that the firms have identical production technologies with a marginal cost of $c = 2$; firms know their own marginal cost but they are not given information about the other firm's marginal cost. This lack of knowledge prevents students from guessing at, and possibly attempting to manipulate, the profits of the other firm, and helps keep the students focused on maximizing their own profits.

The parameters we used are chosen to ensure that:

1. There is a unique equilibrium of the discrete game matrix.

2. The second movers have a unique best response for every quantity the first movers might choose. (Unlike a game with a continuous strategy space, a Cournot game with a discrete strategy set does not necessarily produce a unique best response for second movers.)

3. The second movers' best response function is such that when Firm 1 increases their quantity, Firm 2 chooses to decrease their quantity. Again, in a game with a discrete strategy set, if the products are highly differentiated, Firm 2 might choose the same
strategy in response to multiple strategies of Firm 1. This lack of variation in response would inhibit students from gaining an understanding of strategic interactions and make the game much less interesting.

4. The parameter $\alpha$ is chosen so that the prices are significantly high for the most of the possible outcomes, so that profit amounts are significantly different across the different outcomes; large variations in profits will encourage students to test a variety of strategies.

Instructors can choose any set of parameters that satisfy these requirements and modify the game to accommodate a higher or lower degree of product differentiation, a larger strategy set, or higher profits. An added feature of the parameters we chose is the fact that if both firms choose high quantities, the price level goes down to zero and they end up with losses. This illustrates that when both firms try to sell high quantities the result is losses for both firms. This will encourage the second movers to choose a low quantity whenever the first movers choose a high quantity.

With these parameters, the continuous-space equilibrium strategies would be $q_1 = 15$ and $q_2 = 11$. To keep the experiment manageable and limit the amount of experimenting students have to do to reach an equilibrium, we restrict student strategies to just four possible choices: $\{5, 10, 15, 20\}$. As a result, the students are expected to converge to $q_1 = 15$ and $q_2 = 10$ in the experiment. Our experience suggests that students generally converge to this predicted equilibrium in about ten rounds. Restricting each firm to 30 seconds to make a choice means that this treatment can be run in less than 15 minutes.
Stackelberg Bertrand Model

For the Bertrand game it is useful to rewrite equations (1) and (2) to express the quantity demanded from the two firms as functions of $p_1$ and $p_2$ as follows:

$$q_i = \delta - \rho p_i + \sigma p_j.$$  

Both firms maximize profits, with the second mover treating the choice of the first mover ($p_1$) as given. Again working backwards, Firm 2's profit maximization problem is

$$\max_{p_2} (p_2 - c)(\delta - \rho p_2 + \sigma p_1).$$

Solving this gives Firm 2's best response as a function of $p_1$:

$$p_2(p_1) = \frac{\delta + \rho p_1 + c\sigma}{2\rho}.$$  

Note that Firm 2's best response function is an upward sloping function of $p_1$. In general, a firm's best response function will be upward sloping when the firms are choosing the price for their product. This implies that when one firm chooses a relatively large price, the other firm also will want to choose a relatively large price and vice versa. In this situation, the firm that gets to choose second generally has an advantage in terms of the amount of profit they can make.

Given equation (6), Firm 1 maximizes profit by solving

$$\max_{p_1} (p_1 - c)(\delta - \rho p_1 + \sigma p_2(p_1)).$$

We assume the parameter values $\delta = 102$, $\rho = 5$, $\sigma = 4$. We again assume the two firms have identical production technologies with a marginal cost of $c = 0$ and, as before, that each firm knows its own marginal cost, but not the other firm's. Let 1 be the index for the firm that moves first. Assuming a continuous strategy space, solving the Stackelberg Bertrand game for these
parameters yields equilibrium strategies of $p_1 = 21$ and $p_2 = 18.6$. Similar to the Cournot game, we restrict the students' price choices to a discrete set: \{17, 18, 19, 20, 21\}. With these values, the equilibrium strategies are $p_1 = 21$ and $p_2 = 19$. In our experience, students converge to the equilibrium values in about ten rounds.

The parameters above are chosen to address concerns similar to those discussed earlier for the Cournot game along with one additional concern: In the Bertrand game, the second mover's best response function is such that when Firm 1 decreases their price, Firm 2 will decrease their price as well. Again, with a discrete strategy set, if the products are highly differentiated, Firm 2 might choose the same strategy for multiple strategies of Firm 1. This lack of variation would make the experiment much less interesting and would inhibit its ability to help students understand the implications of strategic interactions.

**Theoretical Comments**

The existence of a first-mover or second-mover advantage depends on whether the best response functions are positively or negatively sloped. Bulow et al. (1985) defines the terms “strategic substitutes” and “strategic complements” as follows:

- Firms' strategies are said to be strategic substitutes if their best response functions are downward sloping.
- Firms' strategies are said to be strategic complements if their best response functions are upward sloping.
Dowrick (1986) proves that when two firms' best response functions are strategic substitutes, then there is a first mover advantage in the game in the sense that the first mover can obtain greater profits than they would if they were moving second in an otherwise identical market. This occurs because a quantity increase by one firm results in it being optimal for the other firm to decrease its quantity and vice versa. The first mover gains by being first to claim how much it will sell. In the first session of our experiment, students see for themselves that first movers in a Stackelberg Cournot game earn higher profits.

Dowrick (1986) also proves that if two firms face similar cost and demand structures and their best response functions are strategic complements, then there is a second mover advantage in the sense that the second mover can obtain greater profits than they would if they were moving first in an otherwise identical market. This occurs because a price decrease by one firm results in it being optimal for the other firm to decrease its price as well. Similarly, if one firm increases its price, it releases the price pressure on the other firm and the other firm responds by increasing the price of its own good. Hence, the second mover gains by being able to choose a price below that of its competitor. In the second session of our experiment, when firms producing substitute goods compete via prices, students observe that second movers obtain higher profits.

RUNNING THE EXPERIMENT

This experiment consists of two parts: first a session in which students participate in a Cournot game (quantity choice) and then a session in which they participate in a Bertrand game (price choice). In the Cournot session, each firm chooses a quantity to produce and the instructor
combines these with the demand functions to determine the market prices of the goods. In the
Bertrand session, each firm chooses the price for their respective good and the instructor
combines these with the demand functions to determine the quantity sold by each firm. This
difference between the strategic choice variables, price versus quantity, has a dramatic effect on
the benefit of being the first mover. This experiment enables the students to experience the
effect first hand.

The experiment requires sufficient copies of the following sheets from the appendices:

- Appendix A: Instruction sheets for the students who are members of a firm;
- Appendix B: Instructions for the student assistants;
- Appendix C: Outcome matrices for each treatment, (not to be given to firm members!);
  Appendix C1 gives the price and profit matrices for the Cournot game and Appendix C2
gives the quantity and profit matrices for the Bertrand game;
- Appendix D: Worksheet for students who are not active participants in a particular
treatment.

The experiment also requires a coin or some other method of randomly choosing which firm will
move first. (This choice is made only once for each treatment; thereafter, the firms keep their
roles as first mover or second mover.) We also suggest having some form of reward to distribute
to the students after the last period of the game. The instructor should choose two or three
students as assistants. These students will help the instructor process the choices of the firms to
determine market demand and the profit for each firm, move information privately between the
firms and the instructor, and post information for all students to see.
The instructor next divides up to 24 students into four groups. We had students simply count off by fours, but other methods of random assignment would work just as well. Designate one group of students as Firm 1, another as Firm 2, a third as Firm 3, and the fourth as Firm 4. Each firm should have from two to six students. Firms 1 and 2 will play the Cournot game and Firms 3 and 4 will play the Bertrand game. In a larger class, the rest of the students will be designated as spectators.

After the firms are formed, each firm is asked to choose a name for itself and to choose one member as the CEO, who will announce the firm's decisions in each period of the game. All firm's names and the names of their CEOs are recorded on the board for all students to see.

**Cournot Game**

The instructor distributes a copy of the student instruction sheet for the game (Appendix A) to each student. The student assistants are given copies of the Assistant Instructions (Appendix B). All students not in Firm 1 or Firm 2 are given copies of the Outcome Matrices for the Cournot game (Appendix C1) and the spectator handout (Appendix D). The students in each firm should construct a record sheet on which they can keep track of their choices, the other firm’s choices, their market price, and their profits. Students who are not participating directly in the game are asked to observe the experiment closely and to report on the experiment using the questions on the spectator handout. We found that giving the spectators a copy of the outcome matrices helps to keep them engaged and interested and even led some students to attempt to forecast the
choices of the firms. It also helps enrich the subsequent discussion because these students have a broader view of the whole experiment.

The Outcome Matrices for the Cournot game (Appendix C1) include a price matrix, which shows the prices the firms will be able to charge for their respective products for each possible combination of strategies, and a profit matrix, which shows the amount of profit each firm will earn for each possible combination of strategies. Each cell in each matrix has two entries, with the first giving the outcome for Firm 1 and the second giving the outcome for Firm 2. Note that members of Firms 1 and 2 should not see these Outcome Matrices.12

The instructor should read the student instructions aloud and should make sure that all students fully understand them. It can help to ask a couple of questions to make sure the students understand their roles. Students should understand that they are members of firms that produce different but substitutable goods, like soft drinks with different flavors or breakfast cereals with different flavors, shapes, or textures. This means that choices made by one firm affect the profit of the other firm. For example, in the Cournot game, if one firm chooses a high quantity, it not only decreases the price that firm receives, but also decreases the price received by the other firm. It should be emphasized to the students that their final rewards will be based only on the profit their own firm makes in the last round of the game. That is, their reward does not in any way depend on the amount of profits earned by the other firm.

Note that the students in the firms do not know the specific demand function for their product; instead, they are given a set of strategies from which to choose. This way, the students can only
infer the demand as they experience the market during the experiment, similar to how a real-life firm would acquire this information. After each firm chooses its strategy, the student assistants compute the market equilibrium (the instructor may need to help with this for the first round or two) and privately report to each firm their selling price (for the Cournot game) or quantity sold (for the Bertrand game) and the profit earned by that firm. It should be made clear to students that they will need to try different strategies to see how their profits are affected by their choices and by the other firm's choices.

Just like real-life markets, classroom experiments of firm interaction are prone to reputation building and collusion. These are strategies that real firms sometimes use to capitalize on the repeated nature of their interactions and opportunities to communicate. Such strategies can interfere with the experiment's success and care should be taken to ensure that students focus on maximizing their own profits and not thinking about the profits of the other firm.

The students in Firm 1 should be separated from those in Firm 2 and both groups should be separated from the other students in the class, perhaps by moving the Firms to different sides of the front of the room. A coin is tossed (or other mechanism) to determine which firm moves first. Suppose Firm 1 moves first. This firm is given 30 seconds to choose their quantity (in larger classes, an additional student assistant could be assigned as a time-keeper). When time is up, the firm's CEO announces their quantity choice. After hearing the choice of the first firm, the second firm is given 30 seconds to deliberate and announce their quantity choice. When the CEO of the second firm announces their choice, each firm records the choices of both firms on their record sheets. A student assistant collects one record sheet from each firm and brings them to
another assistant, who finds the strategy combination selected by the two firms on the Cournot Outcome Matrices. This assistant records the price and profit for each firm on their respective record sheets. Finally, the first student assistant returns the respective record sheets to each firm, making sure not to reveal one firm's information to the other firm. Both firms observe the quantity choices of both firms, the selling price for their own good, and the profit of their own firm; they receive no information about the price and profit of the other firm.

The first six rounds of the game are used as practice so students can get a sense of the game and the effect different choices they might make can have on their profits. The performance of a firm during the practice periods does not count toward the final earnings of the firm. In the beginning, students might be reluctant to announce a quantity without having specific information about the market demand; but we find that most groups quickly understand that they are not getting any more information and have to choose a strategy anyway. They then typically choose other strategies to see what happens. If they don't vary their choices much on their own, the instructor can encourage them to pick a diversity of strategies to explore the market and learn the impact different strategies may have on their profits. Allowing several practice rounds encourages this experimentation by ensuring students will not be penalized for earning low profits.

At the end of the practice rounds the instructor should allow a short break (about one minute) for students to consider what they have learned so far. They should think about how their own quantity choice and the other firm's quantity choice can affect their profits. In addition, the members of Firm 1 should think about how Firm 2 responded to their quantity choices. Hopefully, students will develop an understanding of how the different choices they might make
affect their profits and also how the other firm's choices affect their profits. The instructor can ask questions to help students see these connections, such as “What happened to your profits when the other firm chose a larger (or smaller) quantity?”

The instructor then announces that the students will not be told in advance when the game will end, and that they will receive a reward based on the profits they earn in the final round. The students should not be told how long the game will last so that they avoid reputation-building strategies that might result in short-term losses and instead seek to play their best strategy in each round. In the next few rounds, we expect to see the second movers choosing strategies consistent with the best responses implied by the Outcome Matrix in Appendix C1 and the first movers learning how to use information about the responses of the second movers to their advantage. The instructor can choose to end the game whenever she wants. In our experience, the firms converge to the predicted equilibrium quantities within 10 rounds after the practice rounds. That is, first movers choose $q_1 = 15$ and second movers respond with $q_2 = 10$. Once the students reach this equilibrium they continue to choose these quantities, making it a good time to end the game.

To summarize this session, the instructor announces the strategies played in the final round and the profits earned by each firm. The members of each firm can be rewarded for their profits, using whatever reward system the instructor deems appropriate. We find that giving tangible rewards to students helps motivate them to work hard to identify their best strategies and we gave students monetary rewards equal to five percent of the profits their firm earned. Other rewards are also possible, including perhaps candy, school supplies, extra credit points, and so on.

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The instructor should next lead a discussion about the advantage observed in the Cournot game.

The following questions can be used to help students articulate what they learned:

1. Do you think one of the firms had an advantage over the other firm? In what way?
2. To first movers: How did you decide that the quantity you chose was the best one?
3. To the second movers: What was the best strategy if the first movers chose a high quantity? Should you choose a high quantity as well? Or a low quantity? What was the best strategy if the first movers chose a low quantity?
4. If there was a round in which both firms incurred losses, you can ask them to discuss why this happened.
5. Can you come up with examples of a real life situation resembling what just occurred?
6. In a real life market, what can a late comer in the market do about the disadvantage they face? They might come up with ideas like innovation (reduces cost of production), advertisement (increases product differentiation.)

The students should observe that Firm 2 chooses lower quantities than does Firm 1. The instructor can point out that the best response function of the second mover is a downward sloping function of the first mover's choice. (Some students may understand this better if the instructor illustrates this relationship by sketching the best response function on the board.) The first mover can choose any point on this function; by choosing a high quantity they take a larger market share and leave less for the second mover, thus restricting the second mover to sell a lower quantity. The instructor can make the first mover advantage clear by emphasizing that, when playing optimal strategies, the first mover earns profits of 1650, whereas the second mover
earns profits of only 1200. The instructor can then generalize this observation with an explanation of Dowrick’s (1986) result for Stackelberg Cournot games as described earlier in this paper.

**Bertrand Game**

After the discussion of the Cournot game, the students in Firms 1 and 2 should return to their seats and the students in Firms 3 and 4 should be separated from other students and will now play the Bertrand game. If desired, the instructor can ask different students to serve as student assistants; make sure any new assistants receive a copy of the student Assistant Instructions (Appendix B). The students in each firm should construct a record sheet on which they can keep track of their choices, the other firm’s choices, their quantity sold, and their profits. All students not in Firm 3 or Firm 4 are given copies of the Outcome Matrices for the Bertrand game (Appendix C2) and the spectator handout (Appendix D). Students who are not participating directly in this game are asked to observe the experiment closely and to report on the experiment using the questions on the spectator handout.

The Outcome Matrices for the Bertrand game (Appendix C2) include a quantity matrix, which shows the quantity each firm will be able to sell for each possible combination of price strategies, and a profit matrix, which shows the amount of profit each firm will earn for each possible combination of price strategies. As before, the members of Firms 3 and 4 should not see these Outcome Matrices. The instructor should again take some time to make sure the students understand their instructions.
Other than the fact that firms now choose prices rather than quantities, the mechanics of running the Bertrand game are identical to those for the Cournot game and so are not repeated here in detail. The instructor again tosses a coin to decide which firm moves first, allows six rounds for practice followed by a short time for discussion, then runs the game until students converge to an equilibrium. One minor difference is that the student assistant must determine the quantity sold by each firm rather than the price they receive for their good.

After the Bertrand game ends, the instructor should lead a discussion about the apparent advantage enjoyed by the second mover in the Bertrand game using appropriate versions of the discussion questions given above for the Cournot game. In addition, the instructor can ask “Can you think of any strategic situations other than price wars that might have second-mover advantage?” The students might come up with advertising wars (whoever spends more on advertising, gets a larger market share) or other, similar responses. The instructor can point out that this is another situation where being able to observe the rival's strategic choice is likely to be better than moving first.

The students should observe that the second mover chose lower prices compared to the first mover. But if the first mover chooses a higher price, the second mover responds by increasing their price as well. The instructor can point out that the best response function of the second mover is an upward sloping function of the first mover's choice. (It may help to illustrate this by drawing the best response function on the board.) The instructor can generalize this observation
with an explanation of Dowrick's (1986) result for Stackelberg Bertrand games, as described earlier.

**Follow up**

After the experiment (perhaps in a subsequent class period), the instructor can use the following two numerical examples to solidify the concepts and results illustrated by the experiment. To illustrate the Cournot Stackelberg game, we present a mathematical solution to the game the students played. Suppose the demand is given by the functions in equations (1) and (2) with the parameters indicated above for the Cournot game. Firm 2's profit maximization problem becomes

$$\max_{q_2} (342 - 10q_2 - 8q_1 - 2)q_2.$$  

From the first order condition of the profit maximization problem, we get the best response function of Firm 2, as shown in equation (3). Using the specified parameters, this function is

$$q_2 = \frac{340 - 8q_1}{20}. \quad (7)$$

Substituting this into the profit maximization problem of Firm 1 (equation (4)) gives the following maximization problem

$$\max_{q_1} q_1 \left[ 342 - 10q_1 - 8 \left( \frac{340 - 8q_1}{20} \right) - 2 \right]$$

Solving the profit maximization problem of Firm 1 by taking the first order condition yields $q_1 = 15$ and substituting it into equation (7) yields the quantity for the second firm, $q_2 = 11$. 

23
At this point the instructor should emphasize the effect of timing on market shares, namely the larger market share commanded by Firm 1. (Note that the equilibrium strategies obtained in this example are not exactly what the students get in the experiment, where they converge to $q_1 = 15$, $q_2 = 10$. This occurs because we use a limited strategy set in the experiment, for reasons discussed above.) Using these equilibrium quantities, the profits of the two firms are $\pi_1 = 1530$ and $\pi_2 = 1210$, similar to those obtained in the experiment and showing a clear first mover advantage.

To illustrate the Bertrand Stackelberg game, we focus on two substitutable but differentiated goods. The demand for these goods is given by equation (5). Using the parameters indicated earlier for the Bertrand game, Firm 2's profit maximization problem becomes

$$\max_{p_2} (102 - 5p_2 + 4p_1)p_2.$$  

From the first order condition of this profit maximization problem, we get the best response function of Firm 2, as shown by equation (6). Using our specified parameters, this function is

$$p_2 = \frac{102 + 4p_1}{10}.$$  

Substituting this into the profit maximization problem of Firm 1 gives the following maximization problem

$$\max_{p_1} \left[ 102 - 5p_1 + 4 \left( \frac{102 + 4p_1}{10} \right) \right] p_1.$$  

Solving the profit maximization problem of Firm 1 by taking the first order condition yields $p_1 = 21$; substituting this into equation (8) yields the equilibrium price level for the second firm, $p_2 = 18.6$.  

24
The instructor can point out that the equilibrium strategies in this example are very close to those obtained in the experiment. (As before, the differences are due to use of a continuous strategy space here versus a discrete strategy space in the experiment.) Using the prices computed here, the profits of the two firms are $\pi_1 = 1499.4$ and $\pi_2 = 1729.8$, similar to those found in the experiment and showing a clear second-mover advantage.

**EXTENSIONS**

The degree of product differentiation affects firms' profits dramatically. If firms are able to make consumers view their products as very different, they can get close to monopoly profit levels even in a Bertrand competition; this is one reason firms spend large sums of money in advertising and marketing. Emphasizing the role of product differentiation on competition can be another use of this experiment.

The instructor can change the product differentiation variable and run the experiment with highly differentiated products for an extra treatment. This can be done by decreasing $\gamma$ in the Cournot treatment and $\sigma$ in the Bertrand treatment. A high degree of differentiation between the products reduces the effect of each firm's strategy on the other firm and students would observe that the other firm's choices do not affect their profits much. The instructor can then ask the students to come up with pairs of products that have a high level of product differentiation (and/or a low level of product differentiation) and to think about the effects of product differentiation on competition. What kind of strategies might firms use in order to increase the perception of
product differentiation? Using the intuition from their experience, students can guess which treatment had a higher level of product differentiation.

Another extension the instructor can pursue is to solve the version of the Bertrand game where the two firms move simultaneously and discuss the effect of sequentiality in a price war. The Bertrand game matrix can be distributed to the students and the students can work on it in groups. The simultaneous version of the game produces a unique Nash equilibrium of \( \{ 17, 17 \} \) and both firms have profits of 1445. The students will notice that the profits are lower for both firms compared to the profits they make in the Stackelberg version of the game, 1533 and 1729 respectively. The instructor can discuss with the students possible reasons for both firms to be better off in a Stackelberg-Bertrand game compared to a simultaneous one. The students may understand that a simultaneous version of the game is a price war in which both firms decrease the price level as much as they can without incurring losses, in order to increase their market share and hence their profits. However when they move sequentially the second firm has no incentive to undercut the first firm any more than necessary. Hence in the sequential version of the game the firms can maintain higher prices (and profits) compared to the simultaneous version.

Finally, an alternative structure for larger classes could be to have all students participate in a firm and to form pairs of firms that will play the game together. Each pair could first play the Cournot game to its conclusion and then play the Bertrand game. To assist with the logistics, each pair of firms could be assigned a student assistant who would handle the game matrices and give information to each firm in the manner described above for the original game. Each
assistant could be assigned a random stopping time which the assistant keeps private. If an instructor has the class time and computer resources, this could be extended even further and firms could be randomly matched every period of the game. One benefit of random matching of firm pairs would be a substantial reduction in concerns regarding the possibilities of collusion and/or reputation building that arise in the original game. This version would work best with a large number of firms so the probability of rematching is low.

CONCLUSION

Many students struggle to fully understand why the order of moves might give one firm an advantage over another. This classroom experiment allows students to experience the challenge of maximizing profit as a leader or as a follower. Through this experience, students develop a deeper understanding of the situations that determine whether a market leader will have an advantage over competitors or whether it will be at a disadvantage.

NOTES

1 Becker and Watts (2008) suggests this line of thought is one reason economics instructors increasingly utilize classroom experiments.

2 Charles Holt has computerized versions of simultaneous-move Cournot and Bertrand games on his website at http://people.virginia.edu/~cah2k/teaching.html.

3 Also, this assumption is necessary for firm profits to be positive in the Bertrand game.
4 Note that in the experiment, information about market demand is not explicitly given to students. Instead, they must discover this information by observing their earnings as a function of the choices of the two firms.

5 It is important to have MC > 0. Otherwise, students might pursue a strategy of maximum possible production to build reputation without incurring losses.

6 If desired, the instructor can change the demand and marginal cost parameters. One thing to keep in mind is that each demand and marginal cost specification results in a unique equilibrium with desired properties when the choice variable is continuous. However in a discrete game matrix, in order to guarantee a unique equilibrium the instructor may have to introduce a large strategy set which would in turn result in a longer session. A game matrix that has more than five choices could increase the duration of the whole experiment to more than 75 minutes.

7 Note that \( \delta, \rho, \) and \( \sigma \) are simply transformations of \( \alpha, \beta, \) and \( \gamma. \)

8 Again, the instructor can change the demand and marginal cost parameters as she sees fit. The instructor should take care to make the game matrix no larger than 6x6 because larger matrices require significantly more time for students to explore the implications of the different options. If non-zero marginal costs are desired, \( \delta = 103 \) and marginal cost of 1, yields a convenient outcome matrix.

9 We gave each student in each firm a modest amount of money as a reward. Instructors wishing to reduce the amount of money they have to pay out could randomly select just one firm to pay.

10 For those preferring a more structured approach, Barkley et al. (2005) describes several different methods for dividing students into groups.

11 We found that having more than six students in a firm creates problems when they try to make quick collective decisions during the experiment.
This restriction helps to ensure the students focus on maximizing their own profit and do not seek to compete with or manipulate the other firm. In initial trials of this experiment, we gave the students the outcome matrices. A particularly astute student used this information to decrease the other firm's profits by choosing very high quantities in the beginning of the game. Using this strategy, she was able to get the first movers in the Cournot game to choose very small quantities and managed to make higher profits as a Cournot follower. In later versions of the experiment, we prevented students from seeing the other firm's profits and this problem did not arise. If the students cannot observe the profits of other firms, they cannot implicitly collude or use their choices strategically to build reputation.

Using profits from the final round and ignoring firms' performance in previous rounds keeps the game as a one shot Stackelberg game. If the instructor chooses to calculate the earnings taking multiple rounds into account, the incentive structure will be of a repeated game and students might start using reputation-building mechanisms or other strategies that may have different equilibria.

The purpose of giving a reward is to encourage students to perform as well as possible during the experiment. Holt (1999) suggests most students are already sufficiently motivated and that significant rewards typically are not needed. We suggest instructors should evaluate the motivation and interest of their students to determine what sort of incentive they might need.
REFERENCES


Appendix A

INSTRUCTIONS FOR FIRMS

There are two firms in this experiment and they are producing related but differentiated products. In other words, the demand your firm faces – and, therefore, the profit you can earn – is affected by the choices of the other firm. The demand functions and production technologies facing the two firms may be very different. Your marginal cost is your private information – do not share it with the other firm.

Your objective is to maximize your firm’s profit!

1. During each period of the game, you must make a choice: in Cournot sessions this will be a quantity of your firm’s product to offer for sale, in Bertrand sessions this will be the price of your product. Your choice must be from the appropriate set given below. One firm will announce their choice first and the other firm will get to observe this choice before making their own choice.

   For Cournot: Your quantity choice must be from the set: \{5, 10, 15, 20\}.
   For Bertrand: Your price choice must be from the set: \{17, 18, 19, 20, 21\}.

2. Construct a Record Sheet for your firm where you can record both your choice and the other firm’s choice, the amount you can charge (Cournot) or sell (Bertrand) and your firm’s profit. Use rows for the different rounds and allow enough space for 15-20 rounds.

3. An assistant will collect your Record Sheet. Another assistant will write on your Record Sheet the market information and profits you make. The market information in Cournot Sessions will be the price you will receive for your product, and in Bertrand Sessions this will be the quantity you sell. Your Record Sheet will then be returned to you. Keep this information private from the other firm.

4. The first six rounds are for you to practice and gain an understanding of the game and how your choices (and the choices of the other firm) can affect your profit. These rounds will not affect your earnings.

5. After the first six rounds, the game could end after any period. (The instructor will randomly decide when to end the game.) You will receive a share of the profit earned by your firm in the last period of the game.
Appendix B

INSTRUCTIONS FOR THE STUDENT ASSISTANTS

The job of the assistants is to help the Instructor make the experiment run smoothly. They do this primarily by helping the Instructor combine the Firms’ choices to form market demand and then determining each firm’s profits, by transferring information between the firms and the instructor (and ensuring it is kept private during transfers), by helping to enforce time constraints, and by performing any other tasks required by the Instructor during the experiment.

The Price, Profit, and Quantity matrices are not to be shown to the players; they are going to have to learn about it by trying different choices.

In each round, after both Firms make their choice, the Student Assistants will:

1. Collect the Record Sheets.

2. Find the corresponding Prices (for the Cournot game) or Quantities (for the Bertrand game) and Profits using the appropriate matrices from Appendix C and enter the appropriate amounts on each Record Sheet. Note that the firms will face different prices (or quantities) because of product differentiation.

3. Return the Records Sheets to each firm.

4. Write the strategy choice of each firm on the board for all students to see.
### Appendix C1

#### COURNOT PRICE AND PROFIT MATRICES

#### Market Price Matrix

<table>
<thead>
<tr>
<th>Firm 1 Quantity Choice</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm 2 Quantity Choice</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>252 / 252</td>
<td>212 / 202</td>
<td>172 / 152</td>
<td>132 / 102</td>
</tr>
<tr>
<td>10</td>
<td>202 / 212</td>
<td>162 / 162</td>
<td>122 / 112</td>
<td>82 / 62</td>
</tr>
<tr>
<td>15</td>
<td>152 / 172</td>
<td>112 / 122</td>
<td>72 / 72</td>
<td>32 / 22</td>
</tr>
<tr>
<td>20</td>
<td>102 / 132</td>
<td>62 / 82</td>
<td>22 / 32</td>
<td>0 / 0</td>
</tr>
</tbody>
</table>

#### Profit Matrix

<table>
<thead>
<tr>
<th>Firm 1 Quantity Choice</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm 2 Quantity Choice</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>2000 / 1050</td>
<td>1600 / 1600</td>
<td>1200 / 1650</td>
<td>800 / 1200</td>
</tr>
<tr>
<td>15</td>
<td>2250 / 850</td>
<td>1650 / 1200</td>
<td>1050 / 1050</td>
<td>450 / 400</td>
</tr>
<tr>
<td>20</td>
<td>2000 / 650</td>
<td>1200 / 800</td>
<td>400 / 450</td>
<td>-40 / -40</td>
</tr>
</tbody>
</table>

Note: The entries in the first table above are generated from the demand equations:

\[
P_1 = 342 - 10q_1 - 8q_2
\]

and

\[
P_2 = 342 - 10q_2 - 8q_1.
\]

The entries in the second table are generated from the profit equation

\[
\pi_i = q_i(p_i - c).
\]
### Quantity Matrix

<table>
<thead>
<tr>
<th>Firm 1 Price Choice</th>
<th>17</th>
<th>18</th>
<th>19</th>
<th>20</th>
<th>21</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>85 / 85</td>
<td>89 / 90</td>
<td>93 / 75</td>
<td>97 / 70</td>
<td>101 / 65</td>
</tr>
<tr>
<td>18</td>
<td>80 / 89</td>
<td>84 / 84</td>
<td>88 / 79</td>
<td>92 / 74</td>
<td>96 / 69</td>
</tr>
<tr>
<td>19</td>
<td>75 / 93</td>
<td>79 / 88</td>
<td>83 / 83</td>
<td>87 / 78</td>
<td>91 / 73</td>
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<tr>
<td>20</td>
<td>70 / 97</td>
<td>74 / 92</td>
<td>78 / 87</td>
<td>82 / 82</td>
<td>86 / 77</td>
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<tr>
<td>21</td>
<td>65 / 101</td>
<td>69 / 96</td>
<td>73 / 91</td>
<td>77 / 95</td>
<td>81 / 81</td>
</tr>
</tbody>
</table>

### Profit Matrix

<table>
<thead>
<tr>
<th>Firm 1 Price Choice</th>
<th>17</th>
<th>18</th>
<th>19</th>
<th>20</th>
<th>21</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>1445 / 1445</td>
<td>1513 / 1440</td>
<td>1581 / 1425</td>
<td>1649 / 1400</td>
<td>1717 / 1365</td>
</tr>
<tr>
<td>18</td>
<td>1440 / 1513</td>
<td>1512 / 1512</td>
<td>1584 / 1501</td>
<td>1656 / 1480</td>
<td>1728 / 1449</td>
</tr>
<tr>
<td>19</td>
<td>1425 / 1581</td>
<td>1501 / 1584</td>
<td>1577 / 1577</td>
<td>1653 / 1560</td>
<td>1729 / 1533</td>
</tr>
<tr>
<td>20</td>
<td>1400 / 1649</td>
<td>1480 / 1656</td>
<td>1560 / 1653</td>
<td>1640 / 1640</td>
<td>1720 / 1617</td>
</tr>
<tr>
<td>21</td>
<td>1365 / 1717</td>
<td>1449 / 1728</td>
<td>1533 / 1729</td>
<td>1617 / 1720</td>
<td>1701 / 1701</td>
</tr>
</tbody>
</table>

Note: The entries in the first table above are generated from the demand equations

\[ q_1 = 102 - 5p_1 + 4p_2 \]

and

\[ q_2 = 102 - 5p_2 + 4p_1. \]

The entries in the second table are generated from the profit equation

\[ \pi_i = q_i(p_i - c). \]
Appendix D

SPECTATOR HANDOUT

Cournot Game
Answer the first three questions below during the practice rounds. Answer questions four through six after the practice rounds.

1. When the first movers choose a high value, the best response for the second movers is to choose ________________.
2. When the first movers choose a low value, the best response for the second movers is to choose ________________.
3. My prediction for this game is that being a first mover is advantageous/disadvantageous. (Circle one and briefly explain why you chose it.)
4. If you were playing this game and you were the second mover, what would your general strategy be?
5. Looking at the game matrix given to you, can you determine the equilibrium of the game?
6. What would be the equilibrium of this game if the two firms were moving simultaneously?

Bertrand Game
Answer the first three questions below during the practice rounds. Answer questions four through six after the practice rounds.

1. When the first movers choose a high value, the best response for the second movers is to choose ________________.
2. When the first movers choose a low value, the best response for the second movers is to choose ________________.
3. My prediction for this game is that being a first mover is advantageous/disadvantageous. (Circle one and briefly explain why you chose it.)
4. If you were playing this game and you were the second mover, what would your general strategy be?
5. Looking at the game matrix given to you, can you determine the equilibrium of the game?
6. What would be the equilibrium of this game if the two firms were moving simultaneously?