

An OLG Model with Intergenerational Strategic Behavior

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Abstract

This paper describes a model of overlapping generations of three-period-lived consumers in which parents are altruistic towards children and give end-of-life bequests. Children attempt to manipulate the size of the bequest they are given – a form of the Samaritan's Dilemma. The success of this manipulation depends, in part, on the timing of the choices made by different individuals. I present three different possibilities. A numerical example illustrates the differences between these possibilities. I construct computer simulations for each possibility and examine their sensitivity to key parameters. Most significant is the strength of the intergenerational altruism.

This paper is intended only to provide a description of the model, leaving substantive policy analysis to other works.

Keywords: strategic behavior, bequests

JEL Classifications: C73, D58, D64, D91

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1 Introduction

The exact nature of intergenerational relationships remains an open question in economics. Unfortunately, the results of policy analyses can depend on the nature of these relationships. For example, Ricardian equivalence is believed to require an absence of intergenerational strategic behavior. Seater(1993) writes that when strategic behavior is included in parent-child interactions “a debt-for-tax swap alters the threat point of the parents and/or the children and therefore has real effects, negating Ricardian equivalence.” (p. 148). If this is true for lump-sum taxes (required to test Ricardian equivalence) then it is certainly true for more realistic tax schemes as well. However, inclusion of strategic behavior is sporadic in policy analyses.

This paper shows how one form of intergenerational strategic behavior can be incorporated into a dynamic, general equilibrium, model. I model the interaction between parent and child as a form of the Samaritan’s dilemma.¹ The selfish child attempts to manipulate the size of a bequest given him by his altruistic parent.² The child can attempt to elicit as large a transfer as possible from the parent by overconsuming when young, so as to be relatively poor when the parent later chooses a bequest amount.³ The parent faces the same problem as does the good Samaritan: how to help the selfish individual without compromising his own consumption too much.

Some authors have recognized the importance of incorporating strategic behavior in dynamic models with intergenerational transfers. Laitner (1993) describes the difficulty involved with doing so:

¹The Samaritan’s dilemma was first presented by James M. Buchanan (1975). Using this type of framework is not new for policy studies. See Coate (1995), Bruce and Waldman (1991), and Lindbeck and Weibull (1988) for examples of various applications to policy analysis.

²Bernheim, Shleifer, and Summers (1985) suggest a desire for child-to-parent services (e.g. phone calls, frequent visits, etc.), rather than altruism, motivates parental transfers. The true motivation for parent-to-child transfers remains an open question in economics and is not an issue addressed in this paper. Bernheim (1991) offers additional discussion on this question. Altruism receives substantial attention in the literature, hence it is the motivation used in this paper.

³Some would say a kid who overconsumes when young is just shortsighted. In fact, a kid who knows his parent will not let him suffer with a low utility level is acting rationally by overconsuming in an early period.

Given multi-period overlaps between parents and their adult children ... ‘single-sided altruism’ would potentially complicate the analysis significantly ... a T -period Stackelberg game would emerge – with parents the ‘leaders’ and their children the ‘followers.’ Children might behave strategically – fully consuming their earnings early in marriage, for example, in order to extract large transfers when the arrival of their own children raised their marginal utility of wealth. (p. 70)

Existing general equilibrium studies of intrafamily economic behavior generally avoid strategic behavior. Common techniques include assuming two-sided altruism (e.g. Altig and Davis (1993) and Laitner (1993) and (1992)) and assuming parents make some decisions for their children (e.g. Batina (1987) and Caballé (1995).) Laitner, Altig and Davis, and O’Connell and Zeldes (1993) each consider intergenerational transfers in a general equilibrium environment, and each mentions the potential significance of intergenerational strategic behavior, but chooses to forgo its inclusion in their analysis. Some general equilibrium analyses do incorporate strategic behavior. For example, selfish parents extract larger-than-expected transfers from altruistic children in Veall (1986) and Cremer, Kessler, and Pestieau (1992) constructs a dynamic version of the strategic bequest motive of Bernheim, Shleifer, and Summers (1985). However, the model developed here appears to be a unique application of the Samaritan’s dilemma to parent-to-child altruism.⁴

⁴An example may help to clarify the nature of the interactions studied in this paper.

Consider a parent with a child in his early years of high school. Both parent and child want the child to attend college after high school. Both are thinking about how to finance the cost of attending college. The parent is already saving for this expense and hopes the child will also contribute some funds.

The child must decide how to use income from his part-time job. He must choose between current consumption and saving funds for college. The child knows the parent is already accumulating resources for this expense. The child also places a high value on current consumption. What should he do? By saving he could obtain a better education, but at the cost of current consumption. The parent will then contribute his saved funds. By not saving the child can enjoy increased current consumption and still go to college. However, this choice means he cannot afford as good a college, or as nice a place to live and food to eat, or in some other way will have a lower quality experience.

The child recognizes the value the parent places on having the child obtain a good education. If the child doesn’t save, the parent may be willing to pay more than originally intended in order for the child to obtain a quality education. If so, not saving today may be the child’s optimal choice.

The child’s ability to manipulate the parent depends on the parent’s affinity for the child and on the parent’s wealth and income levels. The child’s interest in being manipulative depends primarily on his substitution rate between current consumption and an additional unit of college education (beyond what the parent’s intended savings will pay for.) Finally, the parent may anticipate the child’s manipulation and prepare for it by saving less (or perhaps even more) than he otherwise might have. These interactions occur repeatedly in the context of an overlapping generations framework.

I consider three distinct specifications of this model. These specifications differ with respect to assumptions made about the timing of choices within the model. The goal is to contrast two specifications that allow potential strategic behavior with a specification commonly used in the literature (which omits strategic behavior).

The first model specification assumes individuals alive in a period choose their respective consumption, savings, and bequest amounts for the period simultaneously, taking the others' choices as fixed. The second specification assumes individuals alive in a period make their respective choices for the period sequentially within the period, with the eldest individuals choosing first and the youngest individuals choosing last. The third assumes individuals choose consumption and savings amounts for all periods of life, and a bequest amount, in their first period of life. The first two of these specifications are referred to as manipulative since a parent's bequest choice depends on his child's actions. The third specification is referred to as non-manipulative since a bequest amount is chosen at the beginning of an individual's life and cannot later be changed.

I construct a computer simulation for each of the three model specifications and present a numerical example to compare the equilibria of each specification. I then evaluate their sensitivity to key parameters. The equilibria are found to be quite sensitive to the value of the intergenerational discount rate. Varying other parameters has little effect on the results.

2 The Model

2.1 The Environment

The basic framework is a model with overlapping generations of three-period lived consumers. Consumers are homogeneous and there is no aggregate or individual uncertainty. Individuals are intertemporally linked by one-sided intergenerational altruism (parent to child). There is no population growth. Each consumer has one child, born at the beginning of the parent's second period of life. A parent may transfer any nonnegative amount of resources to his child in the second of these overlapping periods.⁵ Only when the individuals are both alive for two overlapping periods can both make potentially manipulative choices in one period while still having a subsequent period of interaction. I assume individuals cannot borrow against possible bequests, a constraint motivated by the fact that, in practice, future bequests are difficult to use as collateral.

The economy has the following additional characteristics:

- A large finite number (N) of identical consumers is born at each time period.
- Each consumer is endowed with one unit of time in each period of life. This time is inelastically supplied as labor.
- A consumer born at time t may save (or borrow) an amount a_j^t at age j ($j = 1, 2$). The net return on saving (cost of borrowing) initiated at time t is r_{t+1} . A consumer may borrow against future wage income, but not against possible bequests.
- There is a government that finances production of a public good (x_t) by collecting per capita lump-sum taxes (τ_t), issuing debt (D_t), or both in each period t . The government maintains a balanced budget in each period:

$$x_t + D_{t-1}(1 + r_t) \leq 3N\tau_t + D_t \quad \forall t = 1, \dots, \infty. \quad (1)$$

⁵In a more general version of this model I allow inter vivos transfers in addition to end-of-life bequests. Because strategic behavior leads consumers to squander resources when young, parents always do best by not giving an inter vivos transfer. Thus prohibiting inter vivos transfers has no effect on the results of this model.

The government's financing decisions and level of public good provision are exogenously specified and are known by all consumers at all times.

- Consumers born at time t have preferences over their own consumption, their child's utility, and the public good as follows:

$$U^t \equiv \left([u(c_1^t) + v(x_t)] + \beta[u(c_2^t) + v(x_{t+1})] + \beta^2[u(c_3^t) + v(x_{t+2})] \right) + \rho U^{t+1} \quad (2)$$

where c_j^t ($j = 1, 2, 3$) is the age j consumption of a consumer born at time t . Assume $u(\cdot)$ is strictly increasing and concave, $\lim_{c \rightarrow 0} u'(c) = \infty$, and $v(\cdot)$ is increasing. $\beta \in (0, 1]$ is the intertemporal discount factor. $\rho \geq 0$ is the intergenerational discount rate.⁶

- The aggregate capital stock is the sum of private and public savings.

$$K_t = Na_1^{t-1} + Na_2^{t-2} - D_t \quad (3)$$

- Production occurs according to the aggregate production function

$$Y_t = F(K_t, L_t) \quad (4)$$

where L_t is aggregate labor supplied at time t . F is strictly increasing and concave with respect to both arguments.

- Prices r_t and w_t are given by the time t marginal products of capital and labor respectively.

Figure 1 shows the structure of a three-period lived overlapping generations model with end-of-life bequests. Superscripts indicate the time of birth of the individual choosing the quantity. The “ a_j 's” give the amount an individual saves from period j to period $j + 1$.

⁶The intergenerational discount rate indicates the weight put on the child's utility in the parent's utility function. For example, a value of 0.5 indicates the parent values his child's utility only 50% as much as the parent values his own utility from consumption. Then, in equilibrium, the parent seeks to equate his child's marginal utility of consumption to twice his own marginal utility of consumption.

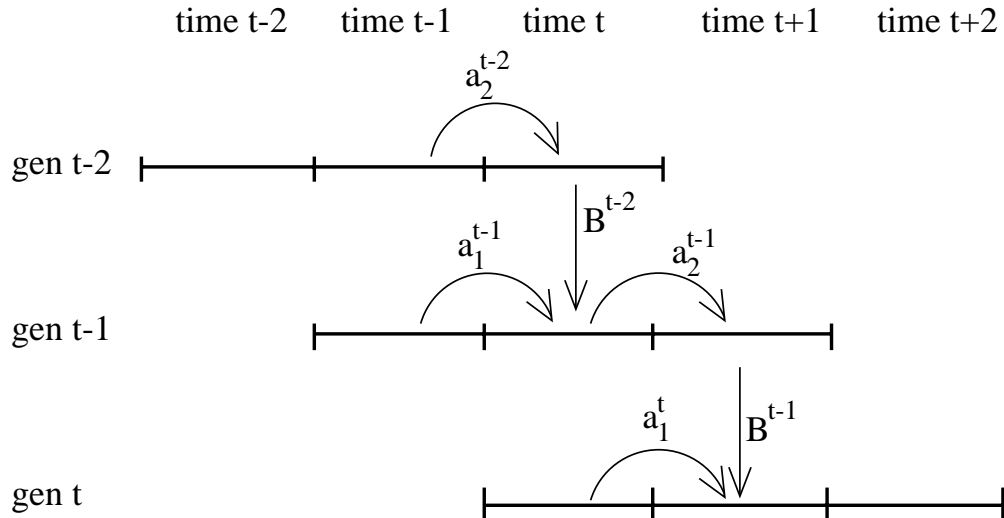


Figure 1: An OLG Model with Three-Period-Lived Consumers

2.2 Manipulation in an Overlapping Generations Model

The next issue to address is specification of when the transfer amount is chosen. Two possibilities exist:

1. The parent chooses a bequest amount at the beginning of his life and is unable to deviate from that choice.⁷
2. The parent chooses the bequest amount in the final period of his lifetime.

To date, researchers using an overlapping generations model have consistently chosen some form of the first approach. We refer to this approach as one of ‘precommitment’ (to the future bequest amount) or as non-manipulative. The bequest amount is chosen during the parent’s first period of life and cannot later be changed. While it is easier to compute, this approach introduces time consistency problems on the part of the parent. Specifically, the parent may wish to change his bequest choice after observing the child’s choices, but is constrained from doing so. Several reasons may lead to a desire to reconsider the bequest choice. For example,

⁷Some authors have chosen a modification of this approach. For example, Caballé (1995), Batina (1987), and Cremer and Pestieau (1993) each use a three-period overlapping generations model and assume a parent makes all of his child’s decisions for the child’s first period of life, thus effectively removing the child’s opportunity for manipulation.

a parent may wish to provide additional resources to a child who squandered resources when young or to give less to a child who saved a large amount when young. The unrealistic nature of this restriction, combined with the time consistency problem, makes precommitment a difficult assumption to defend in practice.⁸

In the second approach the child's first period actions may influence the size of the bequest he receives. This approach gives both individuals the opportunity to behave strategically and is the primary focus of this paper. The strategy available to the child is to overconsume when young (in contrast to smoothing over his lifetime.) Later, when the parent chooses the bequest amount, the child presents himself as a relatively poor individual (poorer than had he smoothed consumption) and asks for a larger bequest. The child's ability to successfully manipulate the parent depends on the parent's affinity for the child and on the parent's wealth and income levels. The child's interest in being manipulative depends primarily on his substitution rate between current and future consumption.

The parent may anticipate the possible manipulation by his child. The strategy available to the parent is to decrease his savings amount in order to reduce his future assets. Fewer parental assets diminishes the child's ability to elicit a larger bequest from the parent.

The parent's success in mitigating the child's potential manipulation depends in part on the timing of their decisions within a period. One possibility is simultaneous choices of consumption, savings and bequests by all consumers alive in a period. A second possibility is sequential choices by the consumers alive in a period: oldest to youngest. When the parent chooses his consumption and savings amounts first he is more successful at reducing the effect of the child's manipulation than he is when their choices are simultaneous. I consider both possibilities in the analysis that follows.⁹ Sections 2.3 and 2.4 describe the details of the different model specifications that arise from the different timing options.

I consider both of these specifications because it is not at all clear one is preferable to the other. Most common in economic literature is to assume simultaneous choices with

⁸Parents may appear to precommit to a bequest amount by writing a will. In fact a parent is also free to subsequently change the will, and thus is not really committing to a bequest amount.

⁹A third possibility exists in that the child could chose before the parent. I omit this case because it seems highly unlikely a parent considering a bequest would wait to make his savings decisions until the child made all of his pre-bequest decisions.

each consumer taking the choices of other consumers as given. However, as O’Connell and Zeldes (1993) point out, “In reality, of course, parents are born before children and make a large fraction of their consumption decisions before their children become independent adults. A more natural modeling approach would therefore be to make parents the ‘leaders’ in a sequential game.”(p.364)

2.3 Simultaneous Choices in a Period

This section describes the model specification arising from the assumption of simultaneous consumption, savings, and bequest choices by the individuals alive in a period and the resulting equilibrium.

Since the amount of the public good is exogenously specified and enters the utility function in an additively separable manner it has no effect on the consumption decisions of consumers. Therefore, for expositional clarity, I omit from the descriptions below.

2.3.1 The Elderly Consumer

The elderly consumer at time t (born at time $t - 2$) chooses consumption, $c_3^{t-2} \in \mathcal{R}_+$, and a bequest, $B^{t-2} \in \mathcal{R}_+$, to solve the following problem:

$$\max_{B^{t-2}, c_3^{t-2}} \left[\beta^2 u(c_3^{t-2}) + \rho \left[\left(\beta u(c_2^{t-1}) + \beta^2 u(c_3^{t-1}) \right) + \rho U^t \right] \right] \quad (5)$$

subject to

$$c_3^{t-2} + B^{t-2} \leq w_t + a_2^{t-2}(1 + r_t) - \tau_t \quad (6)$$

$$\text{and } c_3^{t-2}, B^{t-2} \geq 0$$

where $r_t, w_t, a_2^{t-2}, c_2^{t-1}, a_2^{t-1}, a_1^{t-1}, \tau_t$, and equation (10) are taken as given.

Equation (6) is the elderly consumer’s budget constraint.

The elderly consumer's first order conditions can be combined to give¹⁰

$$\rho u'(c_2^{t-1}) - \beta u'(c_3^{t-2}) = 0. \quad (7)$$

Substituting in the respective budget constraints allows equation (7) to provide a specification of the parent's bequest as a function of the parent's second period savings and his child's first period savings. That is,¹¹

$$B^{t-2} \equiv B(a_2^{t-2}, a_1^{t-1}). \quad (8)$$

The young consumer considers this bequest function when attempting to manipulate the size of his parent's bequest.

2.3.2 The Middle-Aged Consumer

The middle-aged consumer at time t (born at time $t - 1$) chooses consumption, $c_2^{t-1} \in \mathcal{R}_+$, and savings, $a_2^{t-1} \in \mathcal{R}$, to solve the following problem:

$$\max_{a_2^{t-1}, c_2^{t-1}} \left[\left(\beta u(c_2^{t-1}) + \beta^2 u(c_3^{t-1}) \right) + \rho U^t \right] \quad (9)$$

subject to

$$c_2^{t-1} + a_2^{t-1} \leq w_t + a_1^{t-1}(1 + r_t) + B^{t-2} - \tau_t \quad (10)$$

$$\text{and } c_2^{t-1} \geq 0$$

where $r_t, w_t, a_1^{t-1}, B^{t-2}, a_1^t, c_1^t, \tau_t$, and equation (12) are taken as given.

Equation (10) is the middle-aged consumer's budget constraint.

¹⁰By assumption we are interested only the case of positive bequests. Then, since individuals can borrow or save and $\lim_{c \rightarrow 0} u'(c) = \infty$, all first order conditions are satisfied with equality.

¹¹The appendix gives the specific derivation of this function for the computer simulations.

2.3.3 The Young Consumer

The young consumer, born at time t , chooses consumption, $c_1^t \in \mathcal{R}_+$ and savings, $a_1^t \in \mathcal{R}$, to solve the following problem:

$$\max_{c_1^t, a_1^t} [u(c_1^t) + \beta u(c_2^t) + \beta^2 u(c_3^t) + \rho U^{t+1}] \quad (11)$$

subject to

$$c_1^t + a_1^t \leq w_t - \tau_t \quad (12)$$

$$c_2^t + a_2^t \leq w_{t+1} + a_1^t(1 + r_{t+1}) + B^{t-1} - \tau_{t+1} \quad (13)$$

$$\text{and } c_1^t \geq 0$$

where $w_t, a_2^{t-1}, \tau_t, w_{t+1}, r_{t+1}, \tau_{t+1}$, and equation (8) are taken as given.

Equation (12) is the young consumer's budget constraint. Equation (13) is the budget constraint the young consumer will face at time $t + 1$.

The young consumer may use information about his parent's resources (a_2^{t-1}) to attempt to manipulate the bequest his parent will choose next period. His first order conditions can be combined to give¹²

$$-u'(c_1^t) + \beta u'(c_2^t) \left[1 + r_{t+1} + \frac{\partial B^{t-1}}{\partial a_1^t} \right] \leq 0. \quad (14)$$

where $\frac{\partial B^{t-1}}{\partial a_1^t}$ is obtained from equation (8).

2.3.4 Equilibrium

A Simultaneous-Choice equilibrium is sequences of consumption $\{c_1^t, c_2^t, c_3^t\}_{t=1}^\infty$, savings and bequests $\{a_1^t, a_2^t, B^t\}_{t=1}^\infty$, prices $\{r_t, w_t\}_{t=1}^\infty$, and government policy variables $\{\tau_t, D_t, x_t\}_{t=1}^\infty$ that solve the consumers' problems and satisfy all market clearing conditions.

¹²The appendix offers additional information on derivation of this result.

2.4 Sequential Choices in a Period

This section describes the model specification arising from the assumption of sequential consumption, savings, and bequest choices by the individuals alive in a period. Our uncertainty about the true nature of parent-child interactions, coupled with the fact that the sequential choice specification produces different allocations than does the simultaneous choices specification, strongly indicates we should evaluate it as well.

The primary difference between the simultaneous and sequential choice regimes is that here a middle-aged consumer does not treat the young consumer's savings choice as given. This increases the parent's ability to reduce his child's potential manipulative behavior.

A middle-aged parent influences his young child's choices by first examining the problem the young consumer will face when making his decisions later this period. The young consumer's decisions are (in part) a function of the amount the middle-aged consumer chooses to save this period. Combining the young consumer's first order conditions allows us to formulate his savings choice as a function of the savings choice of today's middle-aged consumer. In the simultaneous specification the middle-aged consumer takes the young consumer's savings choice as fixed. Here, in the sequential specification, the young consumer's savings choice function becomes an additional constraint for the middle-aged consumer. Thus, when choosing a savings amount, the middle-aged consumer considers the effect his choice has on the savings choice of the young consumer. This is reflected below in section 2.4.2 by the fact that the middle-aged consumer takes equation (24) as given.

For expositional clarity I again omit the public good from the descriptions below.

2.4.1 The Elderly Consumer

The elderly consumer at time t (born at time $t - 2$), acts first in the period. He chooses consumption, $c_3^{t-2} \in \mathcal{R}_+$, and a bequest, $B^{t-2} \in \mathcal{R}_+$, to solve the following problem:

$$\max_{B^{t-2}, c_3^{t-2}} \left[\beta^2 u(c_3^{t-2}) + \rho \left[\left(\beta u(c_2^{t-1}) + \beta^2 u(c_3^{t-1}) \right) + \rho U^t \right] \right] \quad (15)$$

subject to

$$c_3^{t-2} + B^{t-2} \leq w_t + a_2^{t-2}(1 + r_t) - \tau_t \quad (16)$$

$$\text{and } c_3^{t-2}, B^{t-2} \geq 0$$

where $r_t, w_t, a_2^{t-2}, a_1^{t-1}, \tau_t$, and equation (19) are taken as given.

The elderly consumer's first-order conditions can again be combined to give¹³

$$\rho u'(c_2^{t-1}) - \beta u'(c_3^{t-2}) = 0. \quad (17)$$

As in section 2.3, substituting in the respective budget constraints allows specification of the parent's bequest as a function of the parent's second period savings and his child's first period savings; i.e., a form identical to that given by equation (8). Again a young consumer considers this bequest function when attempting to manipulate the size of his parent's bequest.

2.4.2 The Middle-Aged Consumer

The middle-aged consumer at time t (born at time $t - 1$) acts second in the period. Having received the bequest chosen by his parent, he chooses consumption, $c_2^{t-1} \in \mathcal{R}_+$, and savings, $a_2^{t-1} \in \mathcal{R}$, to solve the following problem:

$$\max_{a_2^{t-1}, c_2^{t-1}} \left[\left(\beta u(c_2^{t-1}) + \beta^2 u(c_3^{t-1}) \right) + \rho U^t \right] \quad (18)$$

subject to

$$c_2^{t-1} + a_2^{t-1} \leq w_t + a_1^{t-1}(1 + r_t) + B^{t-2} - \tau_t \quad (19)$$

$$\text{and } c_2^{t-1} \geq 0$$

where $r_t, w_t, a_1^{t-1}, B^{t-2}, \tau_t$, and equations (21) and (24) are taken as given.

¹³The assumptions again imply all first order conditions are satisfied with equality.

The middle-aged consumer considers the effect his actions have on the consumption and savings choices of the young consumer. Equation (24) reflects the fact that the young consumer conditions his savings choice on the middle-aged consumer's savings choice. The middle-aged consumer may use this knowledge to mitigate the manipulative potential of the young consumer. Inclusion of equation (24) as a constraint in the middle-aged consumer's problem is the main technical difference between the sequential- and simultaneous-choice specifications.

2.4.3 The Young Consumer

The young consumer, born at time t , acts last in the period. He chooses consumption, $c_1^t \in \mathcal{R}_+$, and savings, $a_1^t \in \mathcal{R}$, to solve the following problem:

$$\max_{c_1^t, a_1^t} \left[u(c_1^t) + \beta u(c_2^t) + \beta^2 u(c_3^t) + \rho U^{t+1} \right] \quad (20)$$

subject to

$$c_1^t + a_1^t \leq w_t - \tau_t \quad (21)$$

$$c_2^t + a_2^t \leq w_{t+1} + a_1^t(1 + r_{t+1}) + B^{t-1} - \tau_{t+1} \quad (22)$$

$$\text{and } c_1^t \geq 0$$

where $w_t, a_2^{t-1}, \tau_t, w_{t+1}, r_{t+1}, \tau_{t+1}$, and equation (8) are taken as given.

The young consumer considers his parent's savings (a_2^{t-1}) choice when attempting to manipulate the bequest his parent will choose next period. His first order conditions can be combined to give

$$-u'(c_1^t) + \beta u'(c_2^t) \left[1 + r_{t+1} + \frac{\partial B^{t-1}}{\partial a_1^t} \right] \leq 0. \quad (23)$$

Substituting in the respective budget constraints and evaluating $\frac{\partial B^{t-1}}{\partial a_1^t}$ from equation (8) allows specification of the young consumer's savings amount as a function of the middle-aged consumer's savings amount. That is,

$$a_1^t \equiv a_1(a_2^{t-1}). \quad (24)$$

The middle-aged consumer can use this function to determine the effect his savings choice will have on the young consumer's savings choice.¹⁴

2.4.4 Equilibrium

A Sequential-Choice equilibrium is sequences of consumption $\{c_1^t, c_2^t, c_3^t\}_{t=1}^\infty$, savings and bequests $\{a_1^t, a_2^t, B^t\}_{t=1}^\infty$, prices $\{r_t, w_t\}_{t=1}^\infty$, and government policy variables $\{\tau_t, D_t, x_t\}_{t=1}^\infty$ that solve the consumers' problems and satisfy all market clearing conditions.

3 Sensitivity Studies

This section presents the results of two sets of sensitivity studies performed using computer simulations of the different model specifications. All comparisons are for steady state equilibria. The specific functional forms used for these studies are as follows:

- **Utility of Consumption** is given by

$$u(c) + v(x) = \frac{c^\gamma}{\gamma} + x \quad (25)$$

with $\gamma < 1$, $\gamma \neq 0$.

- **Production Function** is a Cobb-Douglas form:

$$F(K_t, L_t) = AK_t^\alpha L_t^{(1-\alpha)} \quad (26)$$

with $0 < \alpha < 1$.

First an arbitrarily chosen set of parameter values is used to compute the equilibrium of each different model specification. The values chosen are given in Table 1 with the following exception. To facilitate comparisons between the specifications the output of the sequential-choices specification was scaled up by setting $A = 2.7162$. This exception occurs because the sequential-choices specification inherently produces a lower output level than do the other two specifications when using the same parameter values.

Table 1: Parameters for Sensitivity Studies

α	0.3
β	0.442
γ	-2
ρ	0.15
A	2.5
τ_t	0
x_t	0

Table 2: Steady State Comparison Across Regimes

	Non-Manipulative	Simultaneous Choices	Sequential Choices
period 1 consumption	0.7022	0.8772	0.8276
period 2 consumption	1.0068	0.9350	0.9554
period 3 consumption	1.4434	1.3404	1.3697
bequest	0.1824	1.4519	1.2173
period 1 savings	0.0334	-0.1416	-0.0920
period 2 savings	0.1335	0.3085	0.2185

Table 2 compares the steady state equilibria of the three specifications. As expected, the parent gives the smallest bequest in the non-manipulative specification. Under simultaneous choices the young consumers achieve the greatest manipulation of their parents: first period consumption is greatest, second period consumption least and bequests are largest. Under sequential choices a parent is able to somewhat mitigate his child's manipulation. This is evidenced by the lower bequest amount than that for simultaneous choices. These results are representative of the many different parameter configurations studied.

The second study examines the sensitivity of equilibria to changes in key parameters. The results of this study are presented in Figures 2 - 4. Unless otherwise indicated, parameter values are as given in Table 1.

In each of Figures 2 - 4 the vertical axis shows the bequest given as a percentage of lifetime wealth. Lifetime wealth (LW) is defined as the amount of wealth that would be

¹⁴The appendix gives additional details on the derivation of equation (24).

available to an individual in their final period of life if no consumption occurred. Specifically,

$$LW = w_1(1+r)^2 + (w_2 + B)(1+r) + w_3. \quad (27)$$

The figures show that the portion of wealth bequeathed is quite sensitive to the intergenerational discount rate but relatively insensitive to changes in the intertemporal discount rate (β) or to the CES utility parameter (γ). The first result is hardly surprising: parents bequeath more of their wealth the more altruistic they are. The second result suggests precise selection of values for β and γ has little effect on the results.

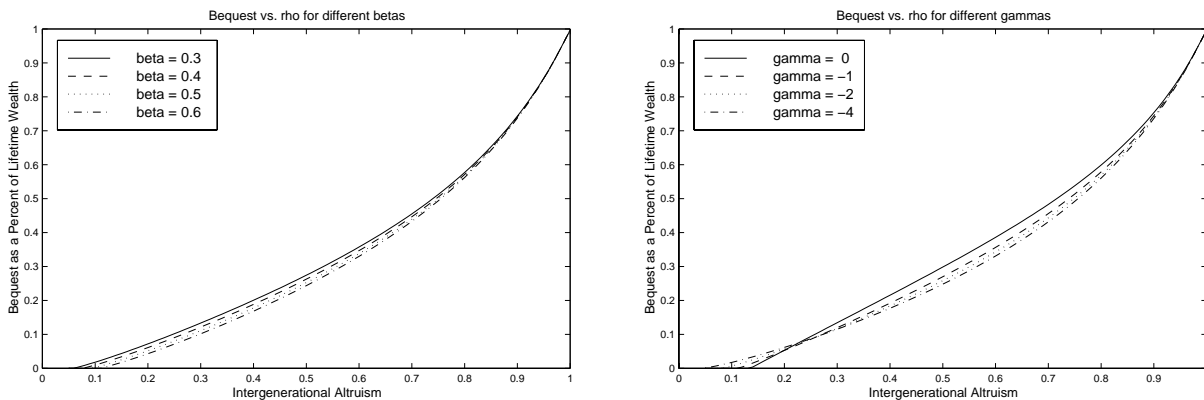


Figure 2: Sensitivity of Simultaneous Choice Equilibrium

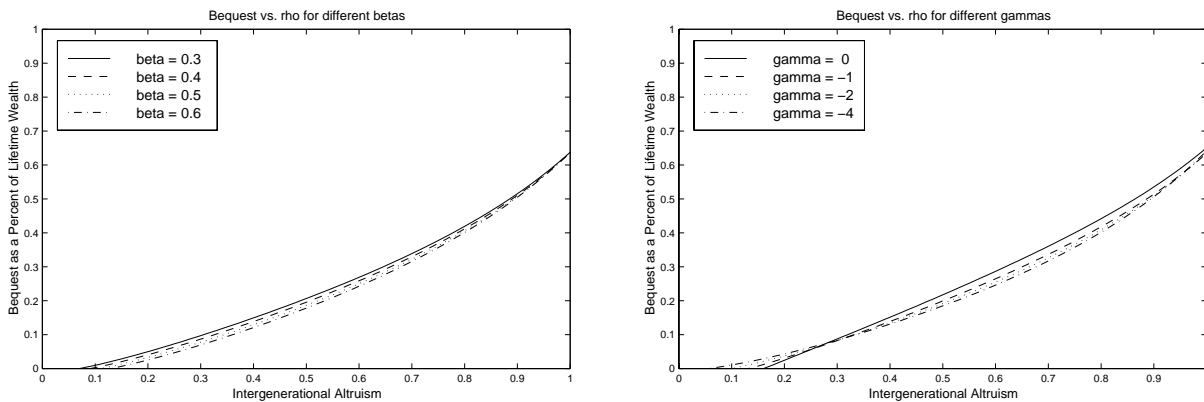


Figure 3: Sensitivity of Sequential Choice Equilibrium

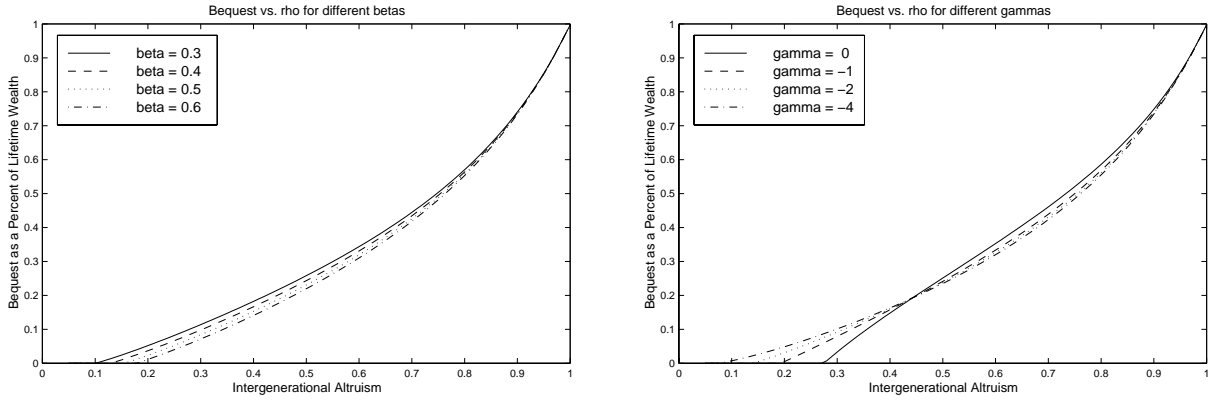


Figure 4: Sensitivity of Precommitment Equilibrium

4 Conclusion

This paper constructs an overlapping generations model with a version of the Samaritan's dilemma. I construct computer simulations to illustrate differences between the possible specifications and examine its sensitivity to key parameters.

We observe that allowing strategic behavior does change the resulting allocations. We also note that non-strategic specifications can suffer time consistency problems. Thus it seems important to continue considering intergenerational strategic behavior in future analyses. Examples can be found in Rebelein (2001a) and (2001b).

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Appendix

Derivation of First Order Conditions

This appendix presents derivations of the consumers' first order conditions and of the specific equations used in the computer simulations. These equations are derived for the sequential choices model specification – the most complex of the three specifications. Comments indicate the modifications required for the simultaneous choices and precommitment specifications.

A.1 The Elderly Consumer

The elderly consumer at time t (born at time $t - 2$) chooses consumption, $c_3^{t-2} \in \mathcal{R}_+$, and a bequest, $B^{t-2} \in \mathcal{R}_+$, to solve the following problem:

$$\max_{B^{t-2}, c_3^{t-2}} \left[\beta^2 u(c_3^{t-2}) + \rho \left[\left(\beta u(c_2^{t-1}) + \beta^2 u(c_3^{t-1}) \right) + \rho U^t \right] \right]$$

subject to

$$\begin{aligned} c_3^{t-2} + B^{t-2} &\leq w_t + a_2^{t-2}(1 + r_t) - \tau_t \\ c_2^{t-1} + a_2^{t-1} &\leq w_t + a_1^{t-1}(1 + r_t) + B^{t-2} - \tau_t \end{aligned}$$

$$\text{and } c_3^{t-2}, B^{t-2} \geq 0$$

where $r_t, w_t, a_2^{t-2}, a_1^{t-1}$, and τ_t are taken as given. (In the simultaneous choice and precommitment specifications c_2^{t-1} and a_2^{t-1} are also taken as given.)

The elderly consumer's first order conditions are most easily evaluated by substituting the budget constraints into his objective function, thereby eliminating the consumption terms, and differentiating with respect to B^{t-2} . This obtains:

$$-\beta^2 u'(c_3^{t-2}) + \rho\beta u'(c_2^{t-1}) \frac{\partial c_2^{t-1}}{\partial B^{t-2}} + \rho\beta^2 u'(c_3^{t-1}) \frac{\partial c_3^{t-1}}{\partial B^{t-2}} + \rho^2 \frac{\partial U^t}{\partial B^{t-2}} \leq 0$$

We next want to determine $\frac{\partial c_2^{t-1}}{\partial B^{t-2}}$, $\frac{\partial c_3^{t-1}}{\partial B^{t-2}}$ and $\frac{\partial U^t}{\partial B^{t-2}}$. One possibility is to consider the effect of B^{t-2} on each savings term of the consumer born at time $t-1$, as well as the effect on *his* bequest choice. Similarly we would need to consider the effect of B^{t-2} on the decisions of the consumer born at time t and so on. Much easier is to apply the envelope theorem, realizing these individuals maximize their choices subject to the choice of B^{t-2} . Then $\frac{\partial c_2^{t-1}}{\partial B^{t-2}} = 1$ and $\frac{\partial c_3^{t-1}}{\partial B^{t-2}} = \frac{\partial U^t}{\partial B^{t-2}} = 0$. This result is consistent with that of other authors.¹⁵

The elderly consumer's first order condition now reduces to

$$\rho u'(c_2^{t-1}) - \beta u'(c_3^{t-2}) \leq 0. \quad (\text{A.1})$$

Substituting in the respective budget constraints, using the CES utility functions, and rearranging gives

$$B^{t-2} \geq \frac{\rho^\sigma (w_t + a_2^{t-2}(1+r_t) - \tau_t) - \beta^\sigma (w_t - a_2^{t-1} + a_1^{t-1}(1+r_t) - \tau_t)}{\rho^\sigma + \beta^\sigma} \quad (\text{A.2})$$

where $\sigma = \frac{1}{1-\gamma}$.

Since consumers are identical, elderly consumers in each time period face a similar bequest function – different only in the time sub- and superscripts.

The young consumer may use his parent's bequest function when attempting to manipulate the size of his parent's bequest.

¹⁵See Altig and Davis (1993) and (1989), O'Connell and Zeldes (1993), and Caballé (1995) for example.

A.2 The Young Consumer

The young consumer, born at time t , chooses consumption, $c_1^t \in \mathcal{R}_+$ and savings, $a_1^t \in \mathcal{R}$, to solve the following problem:

$$\max_{c_1^t, a_1^t} [u(c_1^t) + \beta u(c_2^t) + \beta^2 u(c_3^t) + \rho U^{t+1}]$$

subject to

$$c_1^t + a_1^t \leq w_t - \tau_t$$

$$c_2^t + a_2^t \leq w_{t+1} + a_1^t(1 + r_{t+1}) + B^{t-1} - \tau_{t+1}$$

$$\text{and } c_1^t \geq 0$$

where $w_t, a_2^{t-1}, \tau_t, w_{t+1}, r_{t+1}, \tau_{t+1}$ and equation (A.2) are taken as given. (In precommitment the young consumer also takes B^{t-1} as given.)

The young consumer may use information about his parent's resources (a_2^{t-1}) to attempt to manipulate the bequest his parent will choose next period – hence the inclusion of equation (A.2) as a constraint.

The young consumer's first order conditions are most easily determined by substituting the budget constraints into his objective function, thereby eliminating the consumption amounts, and differentiating with respect to a_1^t . Applications of the envelope theorem give $\frac{\partial c_3^t}{\partial a_1^t} = 0$ and $\frac{\partial U^{t+1}}{\partial a_1^t} = 0$.

We then obtain the following result:

$$-u'(c_1^t) + \beta u'(c_2^t) \left[1 + r_{t+1} + \frac{\partial B^{t-1}}{\partial a_1^t} \right] \leq 0.$$

Evaluate $\frac{\partial B^{t-1}}{\partial a_1^t}$ using equation (A.2), which gives

$$\frac{\partial B^{t-1}}{\partial a_1^t} = \frac{-\beta^\sigma (1 + r_{t+1})}{\rho^\sigma + \beta^\sigma}.$$

Using this result and the CES utility functions, we substitute in the appropriate budget constraints and solve for a_1^t . This gives

$$a_1^t \geq \frac{(w_t - \tau_t)A_t + a_2^t - B^{t-1} - w_{t+1} + \tau_{t+1}}{A_t + 1 + r_{t+1}} \quad (\text{A.3})$$

where

$$A_t = \left[\frac{\beta(1 + r_{t+1})\rho^\sigma}{\rho^\sigma + \beta^\sigma} \right]^\sigma.$$

Note: For the precommitment specification a young consumer's savings choice has no effect on the parent's bequest choice. Thus $\frac{\partial B^{t-1}}{\partial a_1^t} = 0$ and

$$A_t = \left[(1 + r_{t+1})\beta \right]^\sigma.$$

Since consumers are identical, young consumers in each time period face a similar first period savings function – different only in the time sub- and superscripts. The middle-aged consumer may use his child's first period savings function when seeking to minimize the child's manipulative behavior.

A.3 The Middle-Aged Consumer

The middle-aged consumer at time t (born at time $t - 1$) chooses consumption, $c_2^{t-1} \in \mathcal{R}_+$, and savings, $a_2^{t-1} \in \mathcal{R}$, to solve the following problem:

$$\max_{a_2^{t-1}, c_2^{t-1}} \left[\left(\beta u(c_2^{t-1}) + \beta^2 u(c_3^{t-1}) \right) + \rho U^t \right]$$

subject to

$$c_2^{t-1} + a_2^{t-1} \leq w_t + a_1^{t-1}(1 + r_t) + B^{t-2} - \tau_t$$

$$c_3^{t-1} + B^{t-1} \leq w_{t+1} + a_2^{t-1}(1 + r_{t+1}) - \tau_{t+1}$$

$$c_1^t + a_1^t \leq w_t - \tau_t$$

and

$$c_2^{t-1} \geq 0$$

where $r_t, w_t, a_1^{t-1}, B^{t-2}, \tau_t, r_{t+1}, w_{t+1}, \tau_{t+1}$, and equations (A.2) and (A.3) are taken as given. (In the simultaneous-choice and precommitment specifications c_1^t and a_1^t are also taken as given.)

The middle-aged consumer's first order conditions are most easily determined by substituting the budget constraints into his objective function, eliminating the consumption terms, and differentiating with respect to a_2^{t-1} . Remember, in the sequential choices specification the choice of a_2^{t-1} affects a_1^t and the choice of a_1^t affects B^{t-1} .

We obtain the following first order condition:

$$-\beta u'(c_2^{t-1}) + \beta^2 u'(c_3^{t-1}) \left[1 + r_{t+1} - \frac{\partial B^{t-1}}{\partial a_2^{t-1}} - \frac{\partial B^{t-1}}{\partial a_1^t} \frac{\partial a_1^t}{\partial a_2^{t-1}} \right] + \rho \frac{\partial U^t}{\partial a_2^{t-1}} \leq 0. \quad (\text{A.4})$$

Evaluation of the last term gives

$$\frac{\partial U^t}{\partial a_2^{t-1}} = u'(c_1^t) \left[-\frac{\partial a_1^t}{\partial a_2^{t-1}} \right] + \beta u'(c_2^t) \left[(1 + r_{t+1}) \frac{\partial a_1^t}{\partial a_2^{t-1}} + \frac{\partial B^{t-1}}{\partial a_2^{t-1}} + \frac{\partial B^{t-1}}{\partial a_1^t} \frac{\partial a_1^t}{\partial a_2^{t-1}} \right].$$

Consider the four terms on the right-hand side of this equation. The first, second and fourth terms together equal zero by application of the envelope theorem (from the young consumer's first order condition.)

So equation (A.4) becomes

$$-\beta u'(c_2^{t-1}) + \beta^2 u'(c_3^{t-1}) \left[(1 + r_{t+1}) - \frac{\partial B^{t-1}}{\partial a_2^{t-1}} - \frac{\partial B^{t-1}}{\partial a_1^t} \frac{\partial a_1^t}{\partial a_2^{t-1}} \right] + \rho \beta u'(c_2^t) \frac{\partial B^{t-1}}{\partial a_2^{t-1}} \leq 0. \quad (\text{A.5})$$

Use the elderly consumer's first order condition to apply the envelope theorem again. This eliminates the third and fifth terms (of the five terms) of equation (A.5).

Now evaluate $\frac{\partial B^{t-1}}{\partial a_1^t}$ and $\frac{\partial a_1^t}{\partial a_2^{t-1}}$.

Note: a_1^t is not directly affected by a_2^{t-1} ; instead the effect is through the parent's bequest choice B^{t-1} . So

$$\frac{\partial a_1^t}{\partial a_2^{t-1}} = \frac{\partial a_1^t}{\partial B^{t-1}} \frac{\partial B^{t-1}}{\partial a_2^{t-1}}.$$

Evaluate $\frac{\partial B^{t-1}}{\partial a_1^t}$ and $\frac{\partial B^{t-1}}{\partial a_2^{t-1}}$ using equation (A.2) and evaluate $\frac{\partial a_1^t}{\partial B^{t-1}}$ using equation (A.3).

Now equation (A.5) reduces to

$$-u'(c_2^{t-1}) + \beta u'(c_3^{t-1}) F_t \leq 0 \quad (\text{A.6})$$

where

$$F_t = 1 + r_{t+1} - \frac{\rho^\sigma \beta^\sigma (1 + r_{t+1})^2}{(\rho^\sigma + \beta^\sigma)^2 (A_t + 1 + r_{t+1})}.$$

Note: For the simultaneous choices and precommitment specifications $\frac{\partial a_1^t}{\partial a_2^{t-2}} = 0$. (In these specifications the middle-aged consumer takes a_1^t as given when choosing a savings amount a_2^{t-1} .) For these specifications we have

$$F_t = 1 + r_{t+1}.$$

Using the CES utility functions, substituting in from the budget constraints, and rearranging equation (A.6) gives

$$a_2^{t-1} \geq \frac{\beta^\sigma F_t^\sigma (w_t + a_1^{t-1}(1 + r_t) + B^{t-2} - \tau_t) + B^{t-1} - w_{t+1} + \tau_{t+1}}{1 + r_{t+1}) + \beta^\sigma F_t^\sigma}. \quad (\text{A.7})$$

Thus we have three equations ((A.2), (A.3), and (A.7)) in three unknowns (B^{t-2} , a_1^t , and a_2^{t-1}) that must be satisfied each period. The computer simulations generate a simultaneous solutions to these three equations using an iterative, guess-and-verify approach.