A Static General-Equilibrium Model in which Monopoly is Superior to Competition

CHRISTINA M.L. KELTON
Economics Center for Education and Research
College of Business, University of Cincinnati
Cincinnati, OH, USA

ROBERT P. REBELEIN*
Department of Economics, Vassar College
Poughkeepsie, NY, USA

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Abstract

We study social welfare under monopoly using a version of the Kelton and Wallace (1995) two-good general-equilibrium monopoly production model. Individuals are heterogeneous with respect to preferences. They have identical production technologies and labor resources, but only a fraction of them share a monopoly license to produce one of the goods. Numerical evidence indicates that social welfare is higher under monopoly than under perfect competition for a surprising number of parameter combinations – particularly when productivity is relatively low for the monopolistically produced good. Numerical examples illustrate and contribute to our understanding of this result.

Key words: General equilibrium, Social welfare, Monopoly versus competition.
JEL Codes: D42, D51, D61, L12

*Corresponding author: Robert P. Rebelein, Department of Economics, Vassar College, 124 Raymond Ave., Box 290, Poughkeepsie, NY 12604; email: rebelein@vassar.edu; phone: 845-437-7393; fax: 845-437-7576.
1 Introduction

Monopoly, a standard topic in microeconomic theory, has been studied primarily through a partial-equilibrium lens. The universally familiar Figure 1 depicts the deadweight loss (DWL) associated with a constant-marginal-, constant-average-cost monopolist. The DWL “triangle” represents the loss to the economy from a market’s being organized monopolistically as opposed to competitively. A large empirical literature dating from Harberger (1954) seeks to provide an estimate of aggregate DWL for the economy. Furthermore, this triangle has been tremendously influential on public antitrust policy, particularly in the areas of horizontal mergers, monopolization, and collusion. With some exceptions (such as a cost defense in proposed mergers or an appeal to dynamic efficiency in the granting of monopoly patents), greater deadweight loss is synonymous with reduced social welfare. Finally, this triangle has provided the motivation for numerous empirical studies linking monopolistic market structure with higher prices (see Weiss [1989] for a collection of industry studies of this type), establishing that lower allocative efficiency is associated with monopolistic markets.

Unfortunately, Figure 1 fails to capture all of the static social-welfare effects of monopoly. The higher price consumers pay for the good or service in the monopolized market leads them to substitute into other goods and services. In the market for a substitute product, a simple outward shift of the demand curve leads to an increase in the welfare derived from consumption of that product. The additions to welfare are dispersed, of course, across many markets, but their sum may be substantial. Meanwhile, monopoly shareholder utility is enhanced through additional consumption of all goods. In fact it is possible, as we will show, that the welfare gains from these sources can exceed the deadweight loss in the monopolized market.

Harberger estimated the deadweight loss due to market power for 73 United States manufacturing industries, under the assumption of unitary price elasticity of demand. He found DWL to be a very small percentage of national income. The Cowling and Mueller (1978) study, using firm- rather than industry-level data, found DWL to be much larger—in fact, up to ten times as high as Harberger’s estimates. A discussion of methodological and empirical issues surrounding DWL estimation may be found in Littlechild (1981) and Jenny and Weber (1983). See Waldman and Jensen (2001) for a textbook treatment of DWL and other social-welfare costs of monopoly.
Figure 1: A Monopoly Market

Price

Deadweight Loss

Quantity

$P^*_M$

$P^*_C$

$Q^*_M$

$Q^*_C$

MC

MR
In this paper, we develop a general-equilibrium model of monopoly.\textsuperscript{2} By tracking all the goods and services in the economy, and all consumers and all producers, we can account for all market-structure effects on social welfare. Our model explicitly accounts for monopolist preferences and utility, along with utility of nonmonopolists. Committing ourselves to a particular utility function for the economy’s agents provides clear benefits. First, we have a tractable alternative to the partial-equilibrium framework. Given a set of values for the model’s parameters, we can numerically compute monopoly price, sales, output, and profits. Moreover, we can determine the sensitivity of social welfare to changes in the various parameters. Second, we provide examples of situations in which social welfare under monopoly actually exceeds social welfare under perfect competition. We do not claim that the situation in which monopoly is preferable to competition is common, or even likely, in real markets. We seek only to demonstrate the possibility.

The Environment

The model is a two-good, general-equilibrium model in a production environment.\textsuperscript{3} There is a continuum of individuals. Each individual \( h \) has equivalent mass and is distinguished by his or her utility parameter \( \rho_h \). All individuals have the same amount of labor resource \( l \). Let \( l_{1h} \) and \( l_{2h} \) be the amounts of labor \( h \) devotes to production of good one and good two, respectively, and let \( a_1 \) and \( a_2 \) be the respective technological coefficients for good-one and good-two production. The coefficient \( a_i \) is the rate, measured in units of good \( i \) per unit of labor, at which \( h \) can produce good \( i \). Hence, individual \( h \) produces an amount of good one equal to \( a_1l_{1h} \) and an amount of good two equal to \( a_2l_{2h} \).

\textsuperscript{2}We are not the first to recognize that a partial-equilibrium approach to social welfare under monopoly is incomplete. For example, see Kay (1983), whose early, general-equilibrium study graphically analyzes both multiple monopolistic markets and the role of monopoly in changing commodity and factor prices.

\textsuperscript{3}The model builds on the framework developed by Kelton and Wallace (1995).

\textsuperscript{4}The assumption that all individuals possess identical technologies is a simplifying assumption. It is not needed for proving existence or uniqueness of competitive equilibrium, but is useful for the analysis of monopoly. Situations of one or a few sellers arise naturally if the technologies are such that only a small number of individuals are capable of producing one of the goods.
(good one, good two) consumption pair. The utility for individual $h$ is

$$u_h(c_{1h}, c_{2h}) = (c_{1h} + \sigma)^{\rho_h}(c_{2h})^{(1-\rho_h)},$$

where $\rho_h \in (0, 1)$ varies uniformly across the consumers, and $\sigma$ is an additional preference parameter.\(^5\) We can think of $\sigma$ as indicating the consumers’ benefit from good one; the lower $\sigma$ is, the more the individual benefits from consuming good one. This is in contrast to $\rho_h$, which indicates the need a consumer $h$ experiences for good one. To simplify the analysis, we choose to fix $\sigma$ for all consumers. For example, in considering anti-rejection drugs for transplant patients, there may be a similar benefit across persons (same $\sigma$) but a varying need for them across the population (different $\rho_h$s).

### Competitive Equilibrium

Using good two as the numeraire good, it can be shown that the competitive equilibrium price for good one is $P_{CE} = a_2/a_1$ in this environment. At this price, individuals are indifferent between producing good one and producing good two. Thus, there are many different labor and output allocations that allow the economy to produce and consume its preferred aggregate amounts of good one and good two.

In addition, any competitive allocation can be shown to be Pareto efficient.\(^6\) At the competitive equilibrium, it is impossible to improve the well-being of any individual without making someone worse off.

### Paper Outline

The analysis proceeds as follows. Section 2 develops the two-good general-equilibrium monopoly production model. The monopoly equilibrium is described, and social welfare

\(^5\) We thank Xiangkang Yin (2001a) for suggesting this utility function, which is a generalization of the Cobb-Douglas function. One of the features of (1) is that a consumer will choose not to purchase good one when its price exceeds the individual’s “reservation price” for the good. Note that $u_h(\cdot, \cdot)$ satisfies $u_{h1} > 0$ and $u_{h2} > 0$, and the matrix of second derivatives is negative semi-definite.

\(^6\) See Kelton and Wallace (1995), Chapter 7.
under monopoly is derived. Section 3 compares social welfare under monopoly to welfare under perfect competition. Cases in which monopoly welfare exceeds welfare under competition are examined. Section 4 concludes.

2 Monopoly Equilibrium

A license is required to produce good one. A group of individuals, each denoted $m$ for monopolist, share such a license. We assume they take on contiguous values of $\rho_m \in (0, c]$, where $c < 1$. Thus, $c$ is the fraction of the population that owns shares in the monopoly. Monopolists differ from nonmonopolists in two essential ways. First, each monopolist shares the license to produce good one and receives an equal share of the profits earned by the monopoly. Second, monopolists are able to purchase as much of good one as they want at the marginal production cost. Monopolists may use their own labor to produce good one and may hire any or all of the other individuals to produce good one, paying a wage rate $w$ in units of good two per unit of labor. We assume the labor market is competitive, so that $w = a_2$ in equilibrium. The monopolists own all units of good one that are produced. They may sell good one to nonmonopolists at price $P$ (measured in units of good two per unit of good one). A monopolist’s utility is given by (1) for $h = m$.

The other individuals (the nonmonopolists) behave competitively given the price $P$ and the wage rate $w$. They choose consumption amounts and a time allocation, dividing their labor between working for the monopolists to produce good one and producing good two on their own. In the continuum of consumers, they have $\rho_h$ values that span the interval $(c, 1)$.

With three preference assumptions, it can be shown that a monopoly equilibrium (consisting of an equilibrium consumption, production, and labor allocation; a price; and a wage)

\footnote{Since real-world owners of companies are generally unable to purchase products at cost, we chose to assign the monopolists the lowest values of $\rho_m$. This means they are least likely to need to purchase good one, and reduces any distortion the purchase-at-cost aspect of the model may produce. See Yin (2001b) for a model of shareholder price discounts, in which firm owners can purchase products at below-market prices.}

\footnote{At $w = a_2$ nonmonopolists are indifferent between working for themselves to produce good two or working for the monopoly.}
exists and is unique.\footnote{See Kelton and Wallace (1995). The first two of these assumptions are as follows. (1) There must exist a price, say $P_H$, above which there is no nonmonopolist demand for good one. (2) The total revenue function, defined below for the monopolist, is strictly concave. A third assumption ensures that some trade will occur between the monopolists and nonmonopolists. As a group, the monopolists will choose positive amounts of both goods in equilibrium, as will the nonmonopolists. The equilibrium is unique in the sense of a unique price and unique sales of good one. As is the case for the competitive equilibrium, we have nonsignificant multiplicity of equilibrium labor and output pairs. In fact, even aggregate employment of nonmonopolists is not uniquely determined since monopolists are free to substitute some of their own labor for the labor of non-license holders. The monopoly allocation is not Pareto efficient. One of the necessary conditions for Pareto efficiency is violated since the monopoly price exceeds the marginal rate of transformation of good-two producers.} Monopolists are better off than in the competitive equilibrium, while all others are worse off.

**Affordability**

Nonmonopolist $h$ faces the affordability constraint $Pc_{1h} + c_{2h} \leq wl_{1h} + a_2 l_{2h}$. Since $w = a_2$ in equilibrium, and because the affordability constraint must be satisfied with equality in equilibrium, this reduces to

$$Pc_{1h} + c_{2h} = a_2 l.$$  \hspace{1cm} (2)

A monopolist $m$ faces the affordability condition

$$(a_2/a_1)c_{1m} + c_{2m} = a_2 l + (\pi^*/c),$$  \hspace{1cm} (3)

where $(\pi^*/c)$ is an individual monopolist’s share of the total profits $\pi^*$ generated by the monopoly.

**Nonmonopolist Demand**

Consumer $h$ maximizes utility (equation (1)) subject to his or her affordability constraint (2). The nature of the utility function makes it likely that, at each price $P$, some individuals are willing to purchase good one while others are not. We define $\rho^*(P)$ to be the smallest value of $\rho_h$ for which any nonmonopolist $h$ has positive demand for good one at price $P$. Demand for good one is zero for individuals with $\rho_h < \rho^*(P)$ and positive for individuals...
with \( \rho_h \geq \rho^*(P) \). The individual demands for good one and good two, as functions of the monopoly price \( P \), are given by equations (4) and (5), respectively:

\[
d_{1h}(P) = \begin{cases} 
\rho_h(a_2l + P\sigma)/P - \sigma & \text{for } \rho_h \geq \rho^*(P) \\
0 & \text{for } \rho_h < \rho^*(P), 
\end{cases}
\]

and

\[
d_{2h}(P) = \begin{cases} 
(1 - \rho_h)(a_2l + P\sigma) & \text{for } \rho_h \geq \rho^*(P) \\
a_2l & \text{for } \rho_h < \rho^*(P). 
\end{cases}
\]

An expression for \( \rho^*(P) \) is derived from the first row of equation (4):

\[
\rho^*(P) = \frac{P\sigma}{a_2l + P\sigma}.
\]

Let \( D_1(P) \) be the aggregate nonmonopolist demand for good one at price \( P \). We find \( D_1(P) \) by integrating over those individuals with positive individual demands (those whose utility parameter exceeds \( \rho^*(P) \)), or over the set of all nonmonopolists, whichever is smaller. Hence,

\[
D_1(P) = \int_{\rho = \max(\rho^*, c)}^{1} d_{1h}(P)d\rho = \int_{\rho = \max(\rho^*, c)}^{1} \rho(a_2l + P\sigma)/P - \sigma]d\rho.
\]

When \( \rho^*(P) \geq c \), the solution to this integral is\(^{10}\)

\[
D_1(P) = \frac{(a_2l)^2}{2P(a_2l + P\sigma)}.
\]

**Monopoly Sales, Price, and Profit**

A monopolist \( m \) also maximizes his or her individual utility ((1) for \( h = m \)) subject to his or her affordability condition (3). That condition is based on the monopoly’s selection of profit-maximizing sales of good one and the associated price at which nonmonopolists are willing to purchase that amount, given their aggregate demand in (8).

\(^{10}\)To simplify the analysis we consider only the case of \( \rho^*(P) \geq c \). This assumption has no qualitative effects on our results. (The solution to (7) and later integrals is substantially more complex when we allow \( \rho^*(P) < c \).)
Letting $D_1^{-1}(q)$ be the price at which nonmonopolists demand in total the quantity $q$ of good one, we rewrite equation (8) as

$$q = \frac{a_2 l^2}{2D_1^{-1}(q)(a_2 l + D_1^{-1}(q)\sigma)}.$$  

Solving for $D_1^{-1}(q)$ gives

$$D_1^{-1}(q) = \frac{[a_2 l(\sqrt{q^2 + 2a_2 l^2} - q)]}{2}.$$

(9)

$TR(q) = qD_1^{-1}(q)$ is the total revenue from selling $q$ units of good one:

$$TR(q) = \frac{[a_2 l(\sqrt{q^2 + 2a_2 l^2} - q)]}{2\sigma}.$$

(10)

Defining $MR(q)$ to be $\frac{dTR}{dq}$, we get

$$MR(q) = a_2 l\left(\frac{q + \sigma - \sqrt{q^2 + 2a_2 l^2}}{2\sigma \sqrt{q^2 + 2a_2 l^2}}\right).$$

(11)

Setting $MR(q) = MC(q)$, where $MC(q) = a_2/a_1$, and solving for the optimal sales of the monopolist, $q_1^M$, gives

$$q_1^M = -\sigma + \frac{\sqrt{4a_1 l^4 + 8a_1 l^3 \sigma^2 + 5a_2 l^2 \sigma^2 + a_3 l^2 \sigma^2}}{2(a_1 l + \sigma)}.$$

(12)

Combining equations (9) and (12), and defining the equilibrium monopoly price, $P_M^*$, to be $D_1^{-1}(q_1^M)$, we find that

$$P_M^* = \frac{a_2 l}{2\sigma} \sqrt{\frac{\sqrt{4a_1 l^2(a_1 l + \sigma) + \sigma a_2 l^2 + 2\sigma \sqrt{a_1 l + a_2 l^2}} - a_2 l}{2\sigma}}.$$

(13)

It can be shown that the monopoly price exceeds the competitive equilibrium price ($a_2/a_1$) for all parameter values.

The monopoly’s profits in equilibrium are

$$\pi^* = q_1^M(P_M^* - \frac{a_2}{a_1}).$$

(14)

This profit is divided equally among the owners of the monopoly.

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Note that demand is inversely related to price, i.e., $\partial D_1^{-1}(q)/\partial q < 0$. 

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Monopoly Output and Employment

Whereas \( q^*_1M \) is the monopolist’s profit-maximizing sales, total equilibrium production of good one, \( Q^*_1 \), is \( q^*_1M \) plus the monopolists’ consumption of good one. In other words, production satisfies total demand for good one by both nonmonopolists and monopolist license holders. Subtracting the monopolists’ own production of good one (good one produced with their own labor), enough nonmonopolist labor is hired to produce the remainder of \( Q^*_1 \).

Monopolist Consumption

A monopolist’s utility maximization leads to the preferred consumption pair \( (c^*_1m, c^*_2m) \):

\[
c^*_1m = \begin{cases} 
\rho_m(a_1l + \frac{a_1\pi^*}{a_2c} + \sigma) - \sigma & \text{for } \rho_m \geq \rho^*_m \\
0 & \text{for } \rho_m < \rho^*_m 
\end{cases}
\]

and

\[
c^*_2m = \begin{cases} 
(1 - \rho_m)(a_2l + \frac{\pi^*}{c} + \frac{a_2\sigma}{a_1}) & \text{for } \rho_m \geq \rho^*_m \\
\frac{a_2\sigma}{a_2\sigma + a_1a_2l + a_1(\pi^*/c)} & \text{for } \rho_m < \rho^*_m 
\end{cases}
\]

where \( \rho^*_m \) is the minimum \( \rho_m \) value for which monopolist \( h \) has positive demand for good one. Specifically,

\[
\rho^*_m = \frac{a_2\sigma}{a_2\sigma + a_1a_2l + a_1(\pi^*/c)}. \tag{15}
\]

Comparative Statics 1

This section examines how monopoly price, sales, and profits are affected by changes in the five model parameters: \( a_1, a_2, l, \sigma, \) and \( c \). Table 1 summarizes our results.\(^{12}\)

\(^{12}\)The comparative-static effects were signed using a combination of analytical and numerical techniques.
Table 1: Comparative Statics

<table>
<thead>
<tr>
<th></th>
<th>(a_1)</th>
<th>(a_2)</th>
<th>(l)</th>
<th>(\sigma)</th>
<th>(c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monopoly Price</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>Monopoly Sales</td>
<td>+</td>
<td>0</td>
<td>+</td>
<td>+/-</td>
<td>0</td>
</tr>
<tr>
<td>Total Profits</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>0</td>
</tr>
</tbody>
</table>
On the other hand, an increase in good-two technology, i.e., a rise in \( a_2 \), has no effect on monopoly sales of good one, but does encourage the monopoly to charge a higher price for good one. As the cost of labor rises \( (w = a_2) \), some of the increase gets passed on to nonmonopolists in the form of higher prices. An increase in the labor resource \( l \) increases nonmonopolist income, which leads to an increase in good-one sales as well as to an increase in monopoly profits.

The lower \( \sigma \) is (that is, the greater the benefit consumers get from good one), the higher the price the monopoly can charge for good one. Profits are higher, although monopoly sales could be higher or lower – sales increase for relatively low values of \( \sigma \) and decrease for relatively large values. When \( \sigma \) increases from a low value consumers respond primarily to the lower price and purchase more. When \( \sigma \) increases from a high value consumers respond primarily to the decrease in the benefit obtained from good one purchases and purchase less.

The parameter \( c \) has no effect on price, sales, or on total monopoly profits. Since we consider only the case \( \rho^*(P) \geq c \), small changes in \( c \) have no effect on aggregate nonmonopolist demand for good one. Hence, the monopoly has the same optimal price and sales, and its profits remain the same.

**Social Welfare Under Monopoly**

Aggregate utility under monopoly is computed separately for nonmonopolists and monopolists. First, for nonmonopolists, combining equations (1), (4), and (5) gives nonmonopolist \( h \)'s utility as a function of price:

\[
 u_h(d_1h(P),d_2h(P)) = \begin{cases} 
 \frac{\rho_h(a_2l + P\sigma)}{P}[\rho_h[(1 - \rho_h)(a_2l + P\sigma)]^{(1 - \rho_h)}] & \text{for } \rho_h \geq \rho^*(P) \\
 \sigma^{\rho_h}(a_2l)^{(1 - \rho_h)} & \text{for } \rho_h < \rho^*(P),
\end{cases}
\]

where \( \rho^*(P) \) is as previously defined. We calculate aggregate utility for nonmonopolists, \( W_{M1} \), by integrating over all such individuals in the economy, remembering that nonmonopolists with sufficiently low values of \( \rho_h \) consume only good two. That is,

\[
 W_{M1}(P) = (a_2l) \int_c^{\max(\rho^*(P),c)} \left( \frac{\sigma}{a_2l} \right)^{\rho} d\rho + (a_2l + P\sigma) \int_{\max(\rho^*(P),c)}^1 P^{-\rho} \rho^{\rho}(1 - \rho)^{(1 - \rho)} d\rho
\]
with the first integral equal to zero if \( \rho^*(P) \leq c \). The first integral gives the utility of nonmonopolists who do not consume good one, and the second gives the utility of those who do.\(^{13}\)

For monopolists, we similarly integrate over the individual utilities. The utility of an individual monopolist \( m \) is

\[
u_m(c_{1m}^*, c_{2m}^*) = \begin{cases} 
(a_2l + \frac{\pi^*}{c} + \frac{a_2\sigma}{a_1})\rho_m(1 - \rho_m)(\frac{a_2}{a_1})^{-\rho_m} & \text{for } \rho_m \geq \rho_m^* \\
(a_2l + \frac{\pi^*}{c})(\frac{\sigma}{a_2l + (\pi^*/c)})\rho_m & \text{for } \rho_m < \rho_m^*.
\end{cases}
\]

Monopolists whose utility parameter exceeds \( \rho_m^* \) consume a positive amount of good one, whereas those for whom \( \rho_m < \rho_m^* \) consume none. The monopolists’ aggregate utility is

\[
W_{M2} = (a_2l + \frac{\pi^*}{c}) \int_0^\min(\rho_m^*, c) \left(\frac{\sigma}{a_2l + (\pi^*/c)}\right)^\rho d\rho \\
+ (a_2l + \frac{\pi^*}{c} + \frac{a_2\sigma}{a_1}) \int_{\min(\rho_m^*, c)}^c \rho^\rho(1 - \rho)^{(1-\rho)}(\frac{a_2}{a_1})^{-\rho} d\rho
\]

with the second integral equal to zero if \( \rho_m^* \geq c \).

Social welfare under monopoly, \( W_M \), is the sum of equations (16) and (17), i.e., \( W_M = W_{M1} + W_{M2} \).

3 Welfare and Distributional Analysis

In this section we compare the welfare and distributional effects of monopoly with those of perfect competition. Our general approach is to compare \( W_M \) with social welfare under perfect competition, identifying cases in which the former exceeds the latter. We see how our results vary across different values for the parameters of the model. We also determine how different individuals fare under the two types of market structure.\(^{14}\)

\(^{13}\)Solving the first of these integrals is straightforward. The solution to the second is discussed in the appendix. The second has the same form as the second integral in equations (17) and (18) below.

\(^{14}\)See Bednarek and Pecchenino (2002) for an alternative method for comparing social welfare using a general-equilibrium model. Whereas their method of compensating differentials is meant to quantify the welfare loss under monopoly, our interest is solely in qualitative comparisons between the two types of market organization.
General Welfare Comparison

The formulas developed in Section 2 for individual and aggregate demand also apply under perfect competition, using the result that equilibrium price under competition is $a_2/a_1$. Similarly, the welfare formula developed there still applies, although it is now unnecessary to perform separate computations for the monopolists and nonmonopolists. Social welfare under perfect competition is

$$W_{CE}(P) = (a_2 l) \int_0^{\rho^*_{CE}} \left( \frac{\sigma}{a_2 l} \right) \rho^* d\rho + (a_2 l + P\sigma) \int_{\rho^*_{CE}}^1 \rho^{\rho^*}(1 - \rho)(1 - \rho) d\rho, \quad (18)$$

where $\rho^*_{CE}$ separates individuals in the economy into those with zero demand for good one ($\rho_h < \rho^*_{CE}$) and those with positive demand for good one ($\rho_h \geq \rho^*_{CE}$).\(^{15}\)

In Figure 2 we illustrate the substantial number of cases in which $W_M > W_{CE}$ for a range of values for $a_1$, $a_2$, and $\sigma$.\(^{16}\) The upward sloping lines depict the locus of parameter combinations for which monopoly and perfect competition produce the same social welfare. For high values of $a_2$ and low values of $a_1$ we see that social welfare under monopoly tends to exceed social welfare under perfect competition. In other words, when productivity is high in good-two production relative to good-one production, the welfare gains under monopoly may exceed the welfare losses associated with monopolistic production. We also observe that, when the benefit of good one is high (i.e., $\sigma$ low), monopoly has a better chance of generating higher social welfare than competition.

When $W_M > W_{CE}$, several effects work together to offset the welfare loss experienced by nonmonopolists. First, the higher price for good one encourages nonmonopolists to increase consumption of good two, thereby increasing welfare in that market. Second, the monopoly owners, with their profits, purchase additional units of good two (further increasing welfare

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\(^{15}\)Two problems necessarily limit our welfare analysis. The difficulty in obtaining closed-form solutions to (16), (17), and (18) makes direct analytical comparisons intractable. We approximate the integrals using numerical analysis. Second, since we cannot assign values \textit{a priori} to our model’s parameters, we must work with a range of possible values in welfare evaluation.

\(^{16}\)Values for $l$ and $c$ for Figure 2 were set at 10.0 and 0.05, respectively. The results shown in Figure 2 are fairly insensitive to changes in these values.
Figure 2: When Monopoly is Superior to Competition

\[ a_2 \text{ (Good Two Productivity)} \]

\[ a_1 \text{ (Good One Productivity)} \]

\[ \sigma = 3 \]
\[ \sigma = 10 \]
\[ \sigma = 20 \]

- Competition Better
- Monopoly Better
in that market). Third, the monopoly owners enjoy more of good one than they do in the competitive equilibrium – increasing their welfare from good one consumption.

Comparative Statics 2

Table 2 summarizes our findings of the sensitivities of $W_{CE} - W_M$ to changes in the different parameters. Unfortunately, none of the parameter effects on $W_{CE} - W_M$ is monotonic. The welfare advantage of perfect competition over monopoly may rise or fall with any of $a_1$, $a_2$, $l$, $\sigma$, or $c$.\(^{17}\) Our numerical experiments, however, indicate that the effect of $a_1$ is positive for the great majority of cases. The higher the technological coefficient (productivity) for good one, the greater the welfare benefit of competition, or the more welfare would be lost by having the good-one sector organized monopolistically. The opposite is the case for good-two productivity. At relatively small values of $a_2$, a rise in good-two productivity enhances the benefit of competition relative to monopoly. However, after a certain point, the benefit starts to decline and eventually becomes negative.

At $l$ “large enough,” where large enough depends on the values of the other four parameters, a rise in the labor allocation will cause competition increasingly to outperform monopoly in terms of social welfare. In other words, at some point, $W_{CE} - W_M$ rises with $l$. Our numerical experiments indicate three ranges of effect for $\sigma$. At low values for this parameter, a rise in $\sigma$ suggests the advantage of competition relative to monopoly falls. As individuals benefit less from good one, competition’s advantage falls. At higher ranges for $\sigma$, however, we observe the opposite relationship. As $\sigma$ rises, competition’s advantage rises. Eventually, at very high values for $\sigma$, we return to the original relationship of a falling $W_{CE} - W_M$ with a rising $\sigma$. Finally, an increase in $c$, the fraction of the individuals who share the monopoly license, at very low values of $c$, appears to enhance the welfare advantage of competition. However, at higher levels of $c$, an increase in this parameter decreases the

\(^{17}\)Table 2 lists the direction of parameter effect for the relative size of each parameter. In other words, $-/+\)$ means that, for small values of the parameter, there is a negative effect on $W_{CE} - W_M$, while a positive effect results for larger values of the parameter.
Table 2: Welfare Advantage of Competition

<table>
<thead>
<tr>
<th></th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$l$</th>
<th>$\sigma$</th>
<th>$c$</th>
</tr>
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<tbody>
<tr>
<td>$W_{CE} - W_M$</td>
<td>$-/+$</td>
<td>$+/-$</td>
<td>$-/+</td>
<td>-/+</td>
<td>-/+</td>
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</tbody>
</table>
welfare advantage of competition relative to monopoly. More individuals are benefiting from
a share of the monopoly profits.¹⁸

**Numerical Examples**

Tables 3 and 4 provide details of three specific situations in which $W_M > W_{CE}$. The
parameter values used for each case appear in Table 3, and equilibrium values appear in
Table 4. In Table 4, $q_1$ and $q_2$ indicate aggregate production (and consumption) of goods
one and two, respectively, in the competitive equilibrium. Note that, in the left-hand side of
Table 4, the welfare computations are separated for nonmonopolists ($W_1$) and monopolists
($W_2$) only to facilitate comparisons with these groups’ respective situations under monopoly.
Under monopoly, $P_M$ indicates the price the monopoly charges per unit of good one, $q_{1M}$
indicates the total amount of good one purchased by nonmonopolists, and $q_2$ again indicates
the total amount of good two purchased by both groups.

In each case, we observe a number of expected results: the monopoly equilibrium occurs
at a higher price and lower quantity traded, generates a smaller welfare for nonmonopolists,
and generates a larger welfare for monopolists than under perfect competition. Yet each case
also produces a higher total social welfare under monopoly than under perfect competition.
A significant cause of this increase is likely to be the increase, under monopoly, in aggregate
good two consumption – an average increase of over 70 percent across these three cases.

Also, in the first two cases, comparing $ρ_{CE}^*$ to $c$ reveals that monopolists choose not to
consume any good one under competition. Their need for good one is not sufficiently great
to warrant positive consumption. Under monopoly, since $ρ_m^* < c$, some monopolists purchase
good one. Monopolists gain welfare in both the good-one and good-two markets.

¹⁸Plots showing the relationship between $W_{CE} - W_M$ and model parameters are easily constructed from
the preceding equations.
Table 3: Parameter Values for Three Cases in which Monopoly is Superior to Competition

<table>
<thead>
<tr>
<th>Case</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$l$</th>
<th>$\sigma$</th>
<th>$c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>2.5</td>
<td>10.0</td>
<td>12.0</td>
<td>4.0</td>
<td>0.05</td>
</tr>
<tr>
<td>II</td>
<td>7.0</td>
<td>19.0</td>
<td>7.5</td>
<td>3.0</td>
<td>0.05</td>
</tr>
<tr>
<td>III</td>
<td>7.0</td>
<td>22.0</td>
<td>2.5</td>
<td>1.0</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Table 4: Three Cases in Which Monopoly is Superior to Competition

<table>
<thead>
<tr>
<th></th>
<th>Competitive Equilibrium</th>
<th>Monopoly Equilibrium</th>
<th>Welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_{CE}$</td>
<td>$q_1$</td>
<td>$q_2$</td>
</tr>
<tr>
<td>I</td>
<td>4.0</td>
<td>13.2</td>
<td>67.1</td>
</tr>
<tr>
<td>II</td>
<td>2.7</td>
<td>24.8</td>
<td>75.1</td>
</tr>
<tr>
<td>III</td>
<td>3.1</td>
<td>8.3</td>
<td>29.0</td>
</tr>
</tbody>
</table>
Impact of Monopoly on Individuals

To assess the impact monopoly has on different individuals, Table 5 depicts the welfare effects for individuals with $\rho_h$ parameters varying from $\rho_h = 0.001$ to $\rho_h = 0.90$. Setting $a_1 = 100$, $a_2 = 200$, $l = 50.0$, $\sigma = 20.0$, and $c = 0.02$, we compute equilibrium consumption amounts and utilities for each individual.\(^{19}\) The first column of Table 5 indicates the status of the individual, that is, whether he or she shares the monopoly license. The second column gives the individual’s specific value of $\rho_h$. The remaining columns depict the utility and consumption amounts each individual enjoys under the competitive or monopoly equilibrium, respectively. Note that the utility values should be viewed qualitatively rather than quantitatively. That is, when comparing utility values the focus should be on relative rather than absolute comparisons.

It is clear from Table 5 that those individuals who need good one the most (those with a high $\rho_h$ value) lose the most under monopoly relative to competition. The nonmonopolist with $\rho_h = 0.02$ has a very small drop in utility if the market for good one is monopolized. Furthermore, he or she alters his or her consumption only slightly (dropping from 255 units to zero units of good one and consuming 638 additional units of good two). The individual with $\rho_h = 0.8$, however, experiences a greater than sixfold drop in utility under monopoly as opposed to perfect competition. He or she cuts back substantially on consumption of good one (from 19,950 units to 1,760 units), while consuming more good two (13,881 units rather than 12,625). The monopolist license holders, on the other hand, have a dramatic increase in utility and consumption of both good one and good two under monopoly. For example, the monopolist with $\rho_h = 0.015$ is able to consume 7807 units of good one and 1,322,674 units of good two under monopoly, versus 129 units of good one and 62,178 units of good two under competition.

A lingering concern about our primary result may be the assumption that monopoly

\(^{19}\)These parameter values have no particular significance other than the fact that the resulting equilibria produce a higher social welfare under monopoly than under perfect competition.
Table 5: Individuals Under Monopoly versus Competition

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$a_1 = 50$</th>
<th>$a_2 = 125$</th>
<th>$l = 500$</th>
<th>$\sigma = 250$</th>
<th>$c = 0.02$</th>
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</thead>
<tbody>
<tr>
<td>NM Utility</td>
<td>38380</td>
<td>1205</td>
<td>39585</td>
<td>2.5</td>
<td>12376</td>
</tr>
<tr>
<td>Owner Util.</td>
<td>17273</td>
<td>25539</td>
<td>42812</td>
<td>27.6</td>
<td>1019</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Utilities (thousands of utils)</th>
<th>Consumption Amounts</th>
</tr>
</thead>
<tbody>
<tr>
<td>In CE</td>
<td>In Monopoly</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Type</th>
<th>$\rho_m$</th>
<th>In CE</th>
<th>In Monop.</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$c_1$</th>
<th>$c_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>monopolist</td>
<td>0.001</td>
<td>62.2</td>
<td>1331</td>
<td>0</td>
<td>62500</td>
<td>287</td>
<td>1341474</td>
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<tr>
<td>monopolist</td>
<td>0.005</td>
<td>60.8</td>
<td>1295</td>
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<td>62500</td>
<td>2436</td>
<td>1336102</td>
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<tr>
<td>monopolist</td>
<td>0.010</td>
<td>59.1</td>
<td>1258</td>
<td>3</td>
<td>62494</td>
<td>5121</td>
<td>1329388</td>
</tr>
<tr>
<td>monopolist</td>
<td>0.015</td>
<td>57.6</td>
<td>1225</td>
<td>129</td>
<td>62178</td>
<td>7807</td>
<td>1322674</td>
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<tr>
<td>monopolist</td>
<td>0.019</td>
<td>56.5</td>
<td>1201</td>
<td>230</td>
<td>61926</td>
<td>9955</td>
<td>1317303</td>
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<tr>
<td>nonmonop</td>
<td>0.020</td>
<td>56.2</td>
<td>56.0</td>
<td>255</td>
<td>61862</td>
<td>0</td>
<td>62500</td>
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<tr>
<td>nonmonop</td>
<td>0.050</td>
<td>49.4</td>
<td>47.4</td>
<td>1012</td>
<td>59969</td>
<td>0</td>
<td>62500</td>
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<td>nonmonop</td>
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<td>0</td>
<td>62500</td>
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<td>1</td>
<td>62466</td>
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<tr>
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<td>36.1</td>
<td>27.6</td>
<td>3538</td>
<td>53656</td>
<td>127</td>
<td>58995</td>
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<td>21.7</td>
<td>4800</td>
<td>50500</td>
<td>252</td>
<td>55525</td>
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<tr>
<td>nonmonop</td>
<td>0.300</td>
<td>26.0</td>
<td>13.9</td>
<td>7325</td>
<td>44188</td>
<td>504</td>
<td>48584</td>
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<tr>
<td>nonmonop</td>
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<td>9.4</td>
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<td>37875</td>
<td>755</td>
<td>41644</td>
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<td>nonmonop</td>
<td>0.500</td>
<td>20.0</td>
<td>6.6</td>
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<td>31562</td>
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<tr>
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<td>27762</td>
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<td>20822</td>
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<tr>
<td>nonmonop</td>
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<td>3.0</td>
<td>19950</td>
<td>12625</td>
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<td>13881</td>
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<tr>
<td>nonmonop</td>
<td>0.900</td>
<td>20.0</td>
<td>2.5</td>
<td>22475</td>
<td>6312</td>
<td>2011</td>
<td>6941</td>
</tr>
</tbody>
</table>
owners can purchase good one at cost. This concern may be valid since the discounted price is one source of the increased utility received by monopolists under monopoly. To illustrate that this source is relatively small we recompute consumption and utility amounts for the monopoly owners assuming that they face the monopoly price for good one. Table 6 presents the results. Notice that, while monopolist utility is smaller (25,186 here vs. 25,539 in Table 5), social welfare under monopoly still exceeds that under competition.\footnote{The data in Table 6 are derived using the assumption that the monopolists must pay $P^*_M$ for good one. That is, their budget constraint in (3) becomes $P^*_M c_{1m} + c_{2m} = a_2 l + (\pi^*/c)$. All other aspects of the model remain unchanged.}

## 4 Conclusion

This paper develops a two-good general-equilibrium model in which a small fraction of the population is granted the right to produce one of the goods. We show that the social welfare resulting under such a monopoly can exceed that obtained under perfect competition. This result differs from the standard “competition is always best” partial-equilibrium outcome because our model incorporates the utility of all of the individuals in the economy and all interactions between the markets for good one and good two. Our model is very simplistic. Future work will incorporate the dynamic nature of industrial markets as well as the high fixed costs and scale economies present in many monopoly situations.
Table 6: When Monopolists Face Market Prices

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$a_1 = 50$</th>
<th>$a_2 = 125$</th>
<th>$l = 500$</th>
<th>$\sigma = 250$</th>
<th>$c = 0.02$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NM Utility</td>
<td>38380</td>
<td>1205</td>
<td>39585</td>
<td>2.5</td>
<td>12376</td>
</tr>
<tr>
<td>Monopoly Eqbm.</td>
<td>17273</td>
<td>25186</td>
<td>42459</td>
<td>27.6</td>
<td>1019</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Utilities (thousands of utils)</th>
<th>In CE</th>
<th>In Monop.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td>$\rho_m$</td>
<td>$c_1$</td>
</tr>
<tr>
<td>monopolist</td>
<td>0.001</td>
<td>0</td>
</tr>
<tr>
<td>monopolist</td>
<td>0.005</td>
<td>0</td>
</tr>
<tr>
<td>monopolist</td>
<td>0.010</td>
<td>3</td>
</tr>
<tr>
<td>monopolist</td>
<td>0.015</td>
<td>129</td>
</tr>
<tr>
<td>monopolist</td>
<td>0.019</td>
<td>230</td>
</tr>
</tbody>
</table>
Acknowledgements

We are indebted to Neil Wallace for allowing us to use this model. Any errors are the responsibility of the authors. Comments are welcome and may be directed to the authors at chris.kelton@uc.edu or rebelein@vassar.edu.
References


Appendix

We use Scientific Workplace to approximate a solution for the second integral in (16), (17), and (18). The general form of the integral is

$$\int_a^b P^{-x} x^r (1 - x)^{(1-x)} dx$$

which is equivalent to

$$-\frac{0.9588}{\ln P} (P^{-b} - P^{-a}) - \frac{0.003}{\ln^2 P} (-P^{-b} b \ln P - P^{-b} + P^{-a} a \ln P + P^{-a})$$

$$+ \frac{8.0 \times 10^{-6}}{\ln^3 P} (-2P^{-b} - P^{-b} b^2 \ln^2 P - 2P^{-b} b \ln P + 2P^{-a} + P^{-a} a^2 \ln^2 P + 2P^{-a} a \ln P)$$

$$- \frac{9.0 \times 10^{-9}}{\ln^4 P} (-P^{-b} b^3 \ln^3 P - 3P^{-b} b^2 \ln^2 P - 6P^{-b} b \ln P + P^{-a} a^3 \ln^3 P + 3P^{-a} a^2 \ln^2 P$$

$$+ 6P^{-a} + 6P^{-a} a \ln P) + \frac{5.0 \times 10^{-12}}{\ln^5 P} (-24P^{-b} - 24P^{-b} b \ln P - 12P^{-b} b^2 \ln^2 P - 4P^{-b} b^3 \ln^3 P$$

$$- P^{-b} b^4 \ln^4 P + 24P^{-a} + 24P^{-a} a \ln P + 12P^{-a} a^2 \ln^2 P + 4P^{-a} a^3 \ln^3 P + P^{-a} a^4 \ln^4 P).$$