Econ 209
Fall 2002
Handout # 2

The Poisson Distribution

The Poisson Distribution: Suppose that the probability of an occurrence is the same for any equally sized interval of time or space and is independent of the number of occurrences in any other interval of time or space. Let λ be the mean number of occurrences in a given time or space interval. Let x be the number of occurrences. The it can be shown that x has a Poisson distribution:

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!} \quad \textbf{for } x = 0,1,2,3\ldots\ldots$$

Example I: Suppose that there are an average of 600 hits per hour on a popular web site. What is the probability that in a given minute 20 hits occur. The average number of hits per minute λ is equal to 10.

$$P(x = 20) = \frac{10^{20} e^{-10}}{20!} = .001866$$

Example II: Suppose a switchboard can handle up to 2 calls per minute. Suppose that the switchboard receives 30 calls per hour on the average. What is the probability that in any given minute the switchboard is overloaded. $\lambda = 1/2$ a call per minute.

$$P(x > 2) = 1 - [P(x = 0) + P(x = 1) + P(x = 2)]$$

$$= 1 - \left(\frac{(1/2)^0 e^{-1/2}}{0!} + \frac{(1/2)^1 e^{-1/2}}{1!} + \frac{(1/2)^2 e^{-1/2}}{2!} \right) = 1 - .986 = .014$$

Example III: A disease occurs on the average in 1 person in 100,000. What is the probability that there are 10 cases of the disease in a city of 500000. $\lambda = 5$.

$$P(x = 10) = \frac{5^{10} e^{-5}}{10!} = .0181$$

Note: An interesting feature of the Poisson distribution is that $\mathbf{E}(x) = \lambda$ **and** $\sigma_x^2 = \lambda$

The **exponential distribution** is closely related to the Poisson distribution. Suppose that x is has a Poisson distribution. Let t = the time interval between occurrences. Then the probability density function of t is given by

$$f(t) = \lambda e^{-\lambda t} \quad \lambda > 0,\ t > 0$$

The **cumulative density function** is

$$F(t) = 1 - e^{-\lambda t}$$

The cumulative density function of a random variables give the probability that the random variable is less than or equal to a particular value. For example $F(2) = P(t \le 2)$

Example: Suppose that the average number of cars passing a given spot is 30 car per hour. What is the probability that there is a five minute time interval with no cars. $\lambda = 1/2.$

$$P(t > 5) = 1 - F(5) = e^{-\frac{1}{2}5} = .0821$$