Hints and Answers

Chapter 6

6.2 Show that VWP and IIA together imply WP.

6.5 How would a “lexicographic dictatorship” work?

6.8 For (c), suppose it is. Find a set of profiles that leads to a contradiction. Justify your approach.

6.10 For (a), if \( x^* \gg 0 \) is a WEA, there must exist \( n \) prices \( (p^*_1, \ldots, p^*_n) \) such that every \( (x')^* \) maximizes agent \( i \)'s utility over their budget set. Look at these first-order conditions and remember that the Lagrangian multiplier for agent \( i \) will be equal to the marginal utility of income for agent \( i \) at the WEA, \( \partial v_i(p^*, p - e^i) / \partial y \). Next, note that \( W \) must be strictly concave. Thus, if we have some set of weights \( \alpha^i \) for \( i \in I \) and an \( n \)-vector of numbers \( \theta = (\theta_1, \ldots, \theta_n) \) such that \( \alpha^i \nabla u_i((x')^*) = \theta \) and \( x^* \) satisfies the constraints, then \( x^* \) maximizes \( W \) subject to the constraints. What if we choose the \( \alpha^i \) to be equal to the reciprocal of the marginal utility of income for agent \( i \) at the WEA? What could we use for the vector \( \theta \)? Pull the pieces together.

6.11 For (b), consider this three-person, three-alternative case due to Sen (1970a). First, let \( xP^1yP^1z, zP^2xP^2y, \) and \( zP^3xP^3y \). Determine the ranking of \( x \) versus \( z \) under the Borda rule. Next, let the preferences of 2 and 3 remain unchanged, but suppose those of 1 become \( x \rightarrow z \rightarrow y \). Now consider the same comparison between \( x \) and \( z \) and make your argument.

6.12 First, look at the proof of Arrow’s theorem for a definition of “decisiveness.” Why can’t \( (x, y) \) and \( (z, w) \) be the same pair? If \( x = z \), invoke \( U \) and suppose that \( xP^i y, wP^j x, \) and \( yP^i w \) for all \( i \). Use \( I^* \) and WP to show that transitivity is violated. If \( x, y, z, \) and \( w \) are all distinct, let \( xP^i y, zP^j w, \) and suppose that \( wP^i x \) and \( yP^i z \) for all \( i \). Take it from here.

6.14 For (b) and (c), see Exercise A2.10 for the necessary definition. For (e),

\[
E(w, y) = \left( \sum_{i=1}^{N} \frac{1}{N} \left( \frac{y_i}{\bar{y}} \right)^{\frac{\rho}{\mu}} \right)^{1/\rho}.
\]

6.15 No, no, no, yes.

6.16 No, yes.