Chapter 1

1.2 Use the definitions.

1.4 To get you started, take the indifference relation. Consider any three points $\mathbf{x}^i \in X, i = 1, 2, 3$, where $\mathbf{x}^1 \sim \mathbf{x}^2$ and $\mathbf{x}^2 \sim \mathbf{x}^3$. We want to show that $\mathbf{x}^1 \sim \mathbf{x}^2$ and $\mathbf{x}^2 \sim \mathbf{x}^3 \Rightarrow \mathbf{x}^1 \sim \mathbf{x}^3$. By definition of $\sim, \mathbf{x}^1 \sim \mathbf{x}^2 \Rightarrow \mathbf{x}^1 \succsim \mathbf{x}^2$ and $\mathbf{x}^2 \succsim \mathbf{x}^1$. Similarly, $\mathbf{x}^2 \sim \mathbf{x}^3 \Rightarrow \mathbf{x}^2 \succsim \mathbf{x}^3$ and $\mathbf{x}^3 \succsim \mathbf{x}^2$. By transitivity of $\succsim, \mathbf{x}^1 \succsim \mathbf{x}^2$ and $\mathbf{x}^2 \succsim \mathbf{x}^3 \Rightarrow \mathbf{x}^1 \succsim \mathbf{x}^3$. Keep going.

1.16 For (a), suppose there is some other feasible bundle $\mathbf{x}'$, where $\mathbf{x}' \sim \mathbf{x}^*$. Use the fact that B is convex, together with strict convexity of preferences, to derive a contradiction. For (b), suppose not. Use strict monotonicity to derive a contradiction.

1.22 Use a method similar to that employed in (1.11) to eliminate the Lagrangian multiplier and reduce $(n+1)$ conditions to only n conditions.

1.23 For part (2), see Axiom 5′: Note that the sets $\succsim(\mathbf{x})$ are precisely the superior sets for the function $u(\mathbf{x})$. Recall Theorem A1.14.

1.27 Sketch out the indifference map.

1.28 Set down all first-order conditions. Look at the one for choice of x_0^*. Use the constraint, and find a geometric series. Does it converge?

1.31 Feel free to assume that any necessary derivatives exist.

1.32 Roy's identity.

1.40 Theorem A2.6.

1.45 Euler's theorem and any demand function, $x_i(\mathbf{p}, y)$.

1.46 For part (a), start with the definition of $e(\mathbf{p}, 1)$. Multiply the constraint by u and invoke homogeneity. Let $\mathbf{z} \equiv u\mathbf{x}$ and rewrite the objective function as a choice over $\mathbf{z}$.

1.51 Take each inequality separately. Write the one as

$$\frac{\partial x_i(\mathbf{p}_i, y)/\partial y}{x_i(\mathbf{p}, y)} \leq \frac{\bar{\eta}}{y}.$$

Integrate both sides of the inequality from $\bar{y}$ to y and look for logs. Take it from there.

1.53 For part (b),

$$v(\mathbf{p}, y) = A^* y \prod_{i=1}^{n} p_i^{-\alpha_i},$$

where $A^* = A\Pi_{i=1}^{n} \alpha_i^{\alpha_i}$.

1.59 Use Slutsky.

1.62 No hints on this.

1.65 For (b), u^0 must be $v(\mathbf{p}^0, y^0)$, right? Rewrite the denominator.

1.66 For (a), you need the expenditure function and you need to figure out u^0. For (b), $I = (u^0 - 1/8)/(2u^0 - 1)$. For (c), if you could show that the expenditure function must be multiplicatively separable in prices and utility, the rest would be easy.