Chapter 3

3.14 First find in (f_i/f_j). Then note that $d\ln(x_j/x_i) = (-1)d\ln(x_j/x_i)$.

3.15 For (a), first take logs to get

$$\ln(y) = \frac{1}{\rho}\ln\left(\sum_{i=1}^{n}\alpha_1 x_i^{\rho}\right).$$

Note that $\lim_{\rho\to 0}\ln(y) = 0/0$, so L'Hôpital's rule applies. Apply that rule to find $\lim_{\rho\to 0}\ln(y)$, then convert to an expression for $\lim_{\rho\to 0} y$. Part (b) is tough. If you become exasperated, try consulting Hardy, Littlewood, and Pólya (1934), Theorem 4.

3.18 Just work with the definitions and the properties of the production function.

3.21 For the second part, let $\mathbf{z}^2 = \mathbf{z}^1 + \Delta\mathbf{z}$ for $\Delta\mathbf{z} \geq \mathbf{0}$.

3.30 $c(y) \equiv atc(y)y$.

3.41 Equations (3.3) and (3.4).

3.43 Work from the first-order conditions.

3.48 Define

$$\pi_v(p, \mathbf{w}, \bar{\mathbf{x}}) \equiv \max_{y,\mathbf{x}} py - \mathbf{w}\cdot\mathbf{x} \qquad \text{s.t.} \qquad f(\mathbf{x}, \bar{\mathbf{x}}) \geq y,$$

sometimes called the **variable profit function**, and note that $\pi_v(p, \mathbf{w}, \bar{\mathbf{x}}) = \pi(p, \mathbf{w}, \bar{\mathbf{w}}, \bar{\mathbf{x}}) + \bar{\mathbf{w}}\cdot\bar{\mathbf{x}}$. Note that π_v possesses *every* property listed in Theorem 3.7, and that the partial derivatives of π_v and $\pi(p, \mathbf{w}, \bar{\mathbf{w}}, \bar{\mathbf{x}})$ with respect to p and $\mathbf{w}$ are equal to each other.

3.53 $K^* = (w_f/w_k)(y_1^2 + y_2^2)^{1/2}$.