Chapter 5

5.2 You want the *total* effect, dv/dp_i. Use Roy's identity.

5.10 Don't use fancy math. Just think clearly about what it means to be Pareto efficient and what it means to solve the given set of problems.

5.12 Use x_2 as numeraire. For (b), remember that neither consumption nor prices can be negative.

5.15 Derive the consumers' demand functions.

5.16 The function $u^2(\mathbf{x})$ is a Leontief form.

5.17 The relative price of x_1 will have to be $\alpha/(1-\alpha)$.

5.18 For (a), $\mathbf{x}^1 = (10/3, 40/3)$.

5.19 Calculate $\mathbf{z}(\mathbf{p})$ and convince yourself if $\mathbf{p}^*$ is a Walrasian equilibrium, then $\mathbf{p}^* \gg \mathbf{0}$. Solve the system of excess demand functions.

5.20 For (b), remember that total consumption of each good must equal the total endowment. Suppose that $\bar{p}$ is a market-clearing relative price of good x_1, but that $\bar{p} \neq p^*$. Derive a contradiction.

5.21 See Assumption 5.2 for a definition of strong convexity.

5.24 $(p_y/p_h)^* = 4\sqrt{2}$ and he works an 8-hour day.

5.25 To show proportionality of the gradients, suppose they are not. Let $\mathbf{z} = (\nabla u^i(\bar{\mathbf{x}}^i)/\|\nabla u^i(\bar{\mathbf{x}}^i)\|) - (\nabla u^j(\bar{\mathbf{x}}^j)/\|\nabla u^j(\bar{\mathbf{x}}^j)\|)$, and show that $u^i(\bar{\mathbf{x}}^i + t\mathbf{z})$ and $u^j(\bar{\mathbf{x}}^j - t\mathbf{z})$ are both strictly increasing in t at $t = 0$. You may use the Cauchy-Schwartz inequality here, which says that for any two vectors, $\mathbf{v}$ and $\mathbf{w}$, $\mathbf{v} \cdot \mathbf{w} \leq \|\mathbf{v}\|\|\mathbf{w}\|$, with equality if and only if the two vectors are proportional.

5.27 You might want to sketch it out. For (b), you have to ask yourself why they would want to trade. Make sure that your answer makes sense: Check yourself by comparing how well off each is when he does not trade and when he does trade.

5.31 Look carefully at the proof in the text. Construct the coalition of worst off members of *every* type. Give each coalition member the "average" allocation for his type.

5.32 For (b), translate into terms of these utility functions and these endowments what it means to be (1) "in the box," (2) "inside the lens," and (3) "on the contract curve." For (d), consider the coalition $S = \{11, 12, 21\}$ and find a feasible assignment of goods to consumers that the consumers in S prefer.

5.33 Redistribute endowments equally. This will be envy-free. Invoke Theorem 5.5 and consider the resulting WEA, $\mathbf{x}^*$. Invoke Theorem 5.7. Now prove that $\mathbf{x}^*$ is also envy-free.

5.34 For (b), see the preceding exercise.

5.35 Fair allocations are defined in Exercise 5.33.

5.36 For (a), indifference curves must be tangent and all goods allocated. For (b), not in general.

5.39 Exercises 1.64 and 4.1 [Actually, this problem only tells you half the story. It follows from *Antonelli's theorem* that $\mathbf{z}(\mathbf{p})$ is both independent of the distribution of endowments *and* behaves like a single consumer's excess demand system if and only if preferences are identical and homothetic. See Shafer and Sonnenschein (1982) for a proof.]