# Chapter 6

6.2 Show that *VWP* and *IIA* together imply *WP*.

6.5 How would a "lexicographic dictatorship" work?

6.8 For (c), suppose it *is*. Find a set of profiles that leads to a contradiction. Justify your approach.

6.10 For (a), if $\mathbf{x}^* \gg \mathbf{0}$ is a WEA, there must exist $n$ prices $(p_i^*, \ldots, p_n^*)$ such that every $(\mathbf{x}^i)^*$ maximizes agent $i$'s utility over their budget set. Look at these first-order conditions and remember that the Lagrangian multiplier for agent $i$ will be equal to the marginal utility of income for agent $i$ at the WEA, $\partial v^i(\mathbf{p}^*, \mathbf{p}^* \cdot \mathbf{e}^i)/\partial y$. Next, note that $W$ must be strictly concave. Thus, if we have some set of weights $\alpha^i$ for $i \in \mathcal{I}$ and an $n$-vector of numbers $\theta = (\theta_1, \ldots, \theta_n)$ such that $\alpha^i \nabla u^i((\mathbf{x}^i)^*) = \theta$ and $\mathbf{x}^*$ satisfies the constraints, then $\mathbf{x}^*$ maximizes $W$ subject to the constraints. What if we choose the $\alpha^i$ to be equal to the reciprocal of the marginal utility of income for agent $i$ at the WEA? What could we use for the vector $\theta$? Pull the pieces together.

6.11 For (b), consider this three-person, three-alternative case due to Sen (1970a). First, let $xP^1yP^1z, zP^2xP^2y$, and $zP^3xP^3y$. Determine the ranking of $x$ versus $z$ under the Borda rule. Next, let the preferences of 2 and 3 remain unchanged, but suppose those of 1 become $x\bar{P}^1z\bar{P}^1y$. Now consider the same comparison between $x$ and $z$ and make your argument.

6.12 First, look at the proof of Arrow's theorem for a definition of "decisiveness." Why can't $(x, y)$ and $(z, w)$ be the same pair? If $x = z$, invoke $U$ and suppose that $xP^ky, wP^jx$, and $yP^iw$ for all $i$. Use $L^*$ and *WP* to show that transitivity is violated. If $x$, $y$, $z$, and $w$ are all distinct, let $xP^ky, zP^jw$, and suppose that $wP^ix$ and $yP^iz$ for all $i$. Take it from here.

6.14 For (b) and (c), see Exercise A2.10 for the necessary definition. For (e),

$$E(w, \mathbf{y}) = \left( \sum_{i=1}^{N} \frac{1}{N} \left( \frac{y^i}{\mu} \right)^{\rho} \right)^{1/\rho}.$$

6.15 No, no, no, yes.

6.16 No, yes.