Andrew:

Seems to me there are a few ways one can do this.

First, there is a direct test for quasiconcavity, originally due to Arrow and Enthoven. It is described in exercise A2.18 in the calculus chapter of the text. (Warning: the first edition had an error in the the statement of that result. The second edition is now error-free on that score). One can find expositions of this result elsewhere, too.

If we have the CES form $u(x_1, x_2) = (x_1^\rho + x_2^\rho)^{1/\rho}$, then the determinants of the bordered Hessian described in exercise A2.18, page 511, of the text, can be reduced to:

$$D_2 = -\left({x_1}^{2\,(-1+\rho)}\,({x_1}^\rho + {x_2}^\rho)^{-2+\frac{2}{\rho}}\right)$$
$$D_3 = -\left((-1+\rho)\;{x_1}^{-2+\rho}\,{x_2}^{-2+\rho}\,({x_1}^\rho + {x_2}^\rho)^{-2+\frac{3}{\rho}}\right)$$

Notice that for $(x_1, x_2) \gg (0, 0)$ and $0 \neq \rho < 1$ we have $D_2 < 0$ and $D_3 > 0$. According to part *ii* of question A2.18 (and as established by Arrow and Enthoven!), the function u is therefore strictly quasiconcave on the strictly positive orthant.

Another way one can go is to notice that the CES form is actually *concave*. You could then apply one of the direct tests for concavity on the function's Hessian. Theorem A2.4, page 467, gives the relationship of the function's concavity to the definiteness of its Hessian; Theorem A2.11 gives a test for definiteness in terms of the signs on the Hessian's principal minors. Here, u is strictly concave on the strictly positive orthant if its Hessian's principal minors alternate in sign, beginning with negative. Then invoke Theorem A1.15, page 448, which establishes that all strictly concave functions are striclty quasiconcave, hence quasiconcave.