Solution to 3.43.

We need to derive Input factor demand functions for x_1 and x_2. We need to assume that we have a profit maxmizing firm. Then

$$\pi = p\, f(x_1, x_2) - w_1 x_1 - w_2 x_2$$

First order conditions:

$$\begin{cases} \pi_1 = p\, g' - w_1 = 0 & (1) \\ \pi_2 = p\, h' - w_2 = 0 & (2) \end{cases}$$

Let's assume:

(1) The determinant of the Jacobian of the system of equations above is nonzero.

(2) $f(x_1, x_2)$ is concave so as to have a maximizer at the solution.

(3) The solution that maximizes π is x_1^* and x_2^*

By Implicit function theorem we can write;

$x_1^*(p, w_1, w_2)$ and $x_2^*(p, w_1, w_2)$.

Let's conduct some comparative static analysis on the way x_1^* and x_2^* respond to variations in w_1 and w_2.

$$\begin{cases} \dfrac{\partial \Pi_1}{\partial w_1} = p \dfrac{\partial x_1^*}{\partial w_1} g'' - 1 = 0 \\ \dfrac{\partial \Pi_2}{\partial w_2} = p \dfrac{\partial x_2^*}{\partial w_2} h'' = 0 \end{cases}$$

$$\Rightarrow \quad p \begin{pmatrix} g'' & 0 \\ 0 & h'' \end{pmatrix} \begin{pmatrix} \partial x_1^* / \partial w_1 \\ \partial x_2^* / \partial w_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\Rightarrow \frac{\partial x_1^*}{\partial w_1} = \frac{1}{p g''} \qquad \frac{\partial x_2^*}{\partial w_1} = 0$$

$$\begin{cases} \dfrac{\partial \Pi_1}{\partial w_2} = p \dfrac{\partial x_1^*}{\partial w_2} g'' = 0 \\ \dfrac{\partial \Pi_2}{\partial w_2} = p \dfrac{\partial x_2^*}{\partial w_2} h'' - 1 = 0 \end{cases}$$

$$\Rightarrow \frac{\partial x_1^*}{\partial w_2} = 0 \qquad \frac{\partial x_2^*}{\partial w_2} = \frac{1}{p h''}$$

For x_1^* and x_2^* to be homogeneous of degree $\frac{1}{2}$ in w, the following Euler's equations should hold:

$$\frac{1}{2} x_1 = w_1 \frac{\partial x_1^*}{\partial w_1} \quad \text{and} \quad \frac{1}{2} x_2 = w_2 \frac{\partial x_2^*}{\partial w_2}$$

$$\Rightarrow \frac{1}{2} x_1 = w_1 \frac{1}{p g''} \qquad \frac{1}{2} x_2 = w_2 \frac{1}{p h''}$$

From first order equation

$$w_1 = p g' \quad \text{and} \quad w_2 = p h'$$

$$\Rightarrow \frac{1}{2} x_1 = \frac{g'}{g''} \quad \text{and} \quad \frac{1}{2} x_2 = \frac{h'}{h''}$$

that we can rewrite as

$$\frac{g''}{g'} = \frac{2}{x_1} \quad \text{and} \quad \frac{h''}{h'} = \frac{2}{x_2}$$

By anti-derivation we get

$$g(x_1) = \frac{1}{3}x_1^3 \quad \text{and} \quad h(x_2) = \frac{1}{3}x_2^3$$

(we consider the constants to be zero throughout the integration process).

Then the conditions we have to impose on $g(x_1)$ and $h(x_2)$ are

$$\boxed{g(x_1) = \frac{1}{3}x_1^3} \quad \text{and} \quad \boxed{h(x_2) = \frac{1}{3}x_2^3}$$

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