Economics 304
Advanced Topics in Macroeconomics

Notes on Intertemporal Consumption Choice

A: The Two-Period Model

Consider an individual who faces the problem of allocating their available resources over two periods of life with the objective of maximizing lifetime utility.

The individual begins the first period with assets A_1. During the period the individual will receive an exogenously determined quantity of income Y_1 and consume an amount C_1. The amount of assets at the end of the period is then $A_1 + Y_1 - C_1$. We allow the individual to borrow if they want to so this quantity is not necessarily positive. The interest rate is r so, after "loaning" this quantity the individual begins the second period with assets $A_2 = (1+r)(A_1 + Y_1 - C_1)$. During the second period the individual will receive an exogenously determined quantity of income Y_2 and consume an amount C_2. The amount of assets at the end of the period is then $A_2 + Y_2 - C_2$. We require this quantity to be nonnegative.

The individual has preferences over consumption in the two periods described by the lifetime utility function $V(C_1, C_2) = U(C_1) + \frac{1}{1+\rho}U(C_2)$ where ρ is the subjective rate of time preference. The "instantaneous" utility function, $U(\cdot)$, is increasing and strictly concave.[1]

The individual's optimization problem is thus to maximize

$$V(C_1, C_2) = U(C_1) + \frac{1}{1+\rho}U(C_2)$$

subject to the constraints

$$\begin{gathered} A_2 = (1+r)(A_1 + Y_1 - C_1) \\ A_2 + Y_2 - C_2 \geq 0 \\ A_1, Y_1, Y_2 \text{ given} \qquad . \end{gathered}$$

To solve this problem observe that the assumption that $U(\cdot)$ is increasing implies that the individual will consume all available resources in the second period so that $C_2 = A_2 + Y_2$.

[1] "Increasing" means that $U' > 0$ and "strictly concave" means that $U'' < 0$. We also impose the condition $U'(0) = \infty$ to prevent zero consumption in any period. This form of the lifetime utility function is called "separable" and it is not without loss of generality. The implication is that the amount of consumption in any period does not affect the marginal utility of consumption in any other period. We will use it throughout this course as it is somewhat of an industry standard due to the simplifications that it brings to the analysis.

Using the first constraint, this can be written as $C_2 = (1+r)(A_1 + Y_1 - C_1) + Y_2$ which can be substituted into the objective to give

$$J(C_1) = U(C_1) + \frac{1}{1+\rho} U((1+r)(A_1 + Y_1 - C_1) + Y_2).$$

This way of writing the problem turns it into a single variable calculus problem so all that we need do is compute $\frac{dJ}{dC_1}$ and set it equal to zero. Doing this, substituting C_2 for $(1+r)(A_1 + Y_1 - C_1) + Y_2$in the result and doing a little algebra yields the condition

$$U'(C_1) = \frac{1+r}{1+\rho} U'(C_2)$$

where $U'(\,\cdot\,)$ is the first derivative of $U(\,\cdot\,)$.

The intuition of this condition is straightforward. Suppose the individual where to reduce consumption in the first period by one unit and make a loan with this unit so that $1+r$ more units of consumption were available in the second period. The loss in utility in the first period would be the marginal utility of consumption in the first period, $U'(C_1)$. the gain in utility in the second period would be $(1+r)U'(C_2)$. The first period value of this gain is $\frac{1+r}{1+\rho} U'(C_2)$. The marginal benefit must equal the marginal loss if the individual is maximizing lifetime utility. This condition and the constraint characterize the individual's optimal consumption choice.

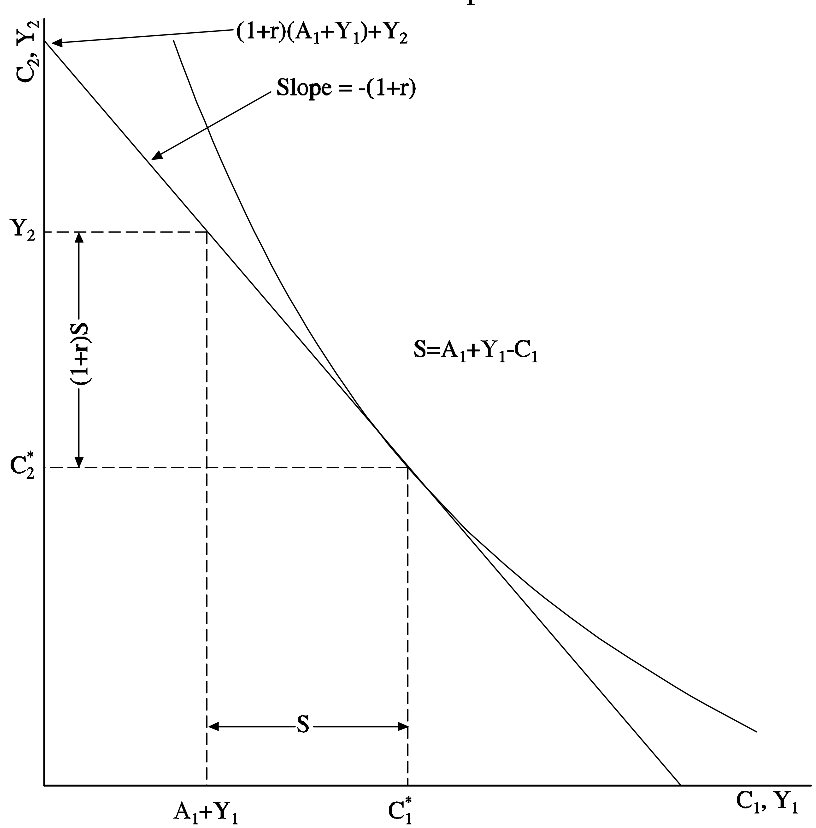
Note how the sign of the slope of the path of consumption over time depends on the relative magnitudes of r and ρ. If, for example, $r > \rho$ then $\frac{1+r}{1+\rho} > 1$ so that $U'(C_1) > U'(C_2)$ implying $C_1 < C_2$. Thus, if the rate of interest is greater (less) than the rate of time preference, consumption will rise (fall) over time. The exact rate of rise or fall depends on the "intertemporal elasticity of substitution" as discussed below.

The model has three principal, not unrelated, implications. The first is that the path of consumption overtime will be "smoother" than that of income in the sense that the change in consumption from one period to the next will tend to be smaller than that in income. This occurs because consumption depends on lifetime wealth, $A_1 + Y_1 + \frac{Y_2}{1+r}$, and not on the current income in the sense that any changes in Y_1 and Y_2 that leave $Y_1 + \frac{Y_2}{1+r}$ unchanged, will not produce changes in consumption. The second implication is that the size of the response of consumption to changes in income depends on whether the change in income is permanent or temporary. A permanent change in income can be modeled as a change in both Y_1 and Y_2 while a temporary change can be modeled as a change in Y_1 alone. Clearly, the former will have a larger effect on C_1 than the latter as the former has a larger effect on lifetime wealth. The third implication is that anticipated changes in income matter for current consumption. An anticipated change in Y_2 will result in a change in C_1.

More generally we can state the broad conclusion of the model as, whenever individuals are given the opportunity to do so, they will attempt to spread out over time the influence of any shocks that would otherwise change current consumption.

The individual's choice is illustrated in the diagram below. Diagrams of this type ought to be familiar to you from Economics 200. The budget line is $C_2 = (1+r)(A_1+Y_1) + Y_2 - (1+r)C_1$ which is a straight line with intercept $(1+r)(A_1+Y_1)+Y_2$ and slope $-(1+r)$ as shown. The individual's indifference curves have the familiar shape and it can be shown that the slope of an indifference curve is given by $\left.\frac{dC_2}{dC_1}\right|_{dV=0} = -(1+\rho)\frac{U'(C_1)}{U'(C_2)}$. The individual achieves the maximal feasible level of utility by choosing that point on the budget line that is also on the highest possible indifference curve. This is the point (C_1^*, C_2^*) as shown. At that point, the budget line and the indifference curve are tangent to each other so that $\left.\frac{dc_2}{dc_1}\right|_{dV=0} = -(1+r)$. Manipulation of the resultant condition gives the condition found above using calculus.

2-Period Consumption Model



B: An Example

Suppose that the instantaneous utility function is $U(C) = \frac{C^{1-\sigma}-1}{1-\sigma}$ with $\sigma > 0$ so that $U'(C) = C^{-\sigma}$. This utility function is called "isoelastic" for reasons that will become clear below. The condition for optimal consumption choice is $C_1^{-\sigma} = \frac{1+r}{1+\rho}C_2^{-\sigma}$. To find the actual optimal quantities of consumption write this condition as which can be written as $C_2 = \left(\frac{1+r}{1+\rho}\right)^{\frac{1}{\sigma}} C_1$. The budget constraint can be written as

$C_2 = (1+r)W - (1+r)C_1$ where $W = A_1 + Y_1 + \frac{Y_2}{1+r}$ is lifetime wealth. Substituting the condition into the constraint and doing a little algebra yields $C_1 = \lambda W$ and $C_2 = (1+r)(1-\lambda)W$ where $\lambda = \frac{1+r}{1+r+\left(\frac{1+r}{1+\rho}\right)^{\frac{1}{\sigma}}}$.

The quantity $\frac{1}{\sigma}$ is the "intertemporal elasticity of substitution" for this utility function. The fact that, in this case, the elasticity does not depend on C is responsible for the name of the utility function. The elasticity measures the willingness of an individual to tolerate changes in their level of consumption over time.[2] To see this write the condition as $\frac{C_2}{C_1} = \left(\frac{1+r}{1+\rho}\right)^{\frac{1}{\sigma}}$ and note that the ratio $\frac{C_2}{C_1}$ is one plus the proportional change in consumption between the two periods. It is clear that this change will be larger the larger is the intertemporal elasticity of substitution. The rise in the ratio due to a higher value of r will also be larger for higher values of the elasticity as the individual responds more to the increased incentive to postpone consumption in the first period.

C: Generalization to Many Periods

We now consider the case of an infinitely-lived individual who receives an exogenously determined amount of income Y_t and consumes an amount C_t in each period of life so that their assets evolve according to $A_{t+1} = (1+r)(A_t + Y_t - C_t)$ for $t = 0, \ldots, \infty$. The initial quantity of assets, A_0, is given. The individual's lifetime utility is given by $V = \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho}\right)^t U(C_t)$ and their objective is to choose a lifetime consumption plan to maximize this subject to the sequence of constraints on the evolution of assets.

To solve this problem write the constraint as $C_t = A_t + Y_t - \frac{A_{t+1}}{1+r}$ and substitute this expression into the objective to get $V = \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho}\right)^t U\left(A_t + Y_t - \frac{A_{t+1}}{1+r}\right)$. Written this way the problem becomes one of choosing a sequence of asset levels to maximize lifetime utility. We can imagine the individual making a plan at $t = 0$ for the rest of their life or imagine that they make a decision each period about how to best allocate their resources between the current and next period knowing that they will make the same decision in all subsequent periods. To solve the latter problem we note that in period t the choice variable is A_{t+1} so we need to compute $\frac{\partial V}{\partial A_{t+1}}$ and set it equal to zero. If we were to write out the objective function and take note of the terms containing A_{t+1}, we would find

$$\ldots + \left(\frac{1}{1+\rho}\right)^t U\left(A_t + Y_t - \frac{A_{t+1}}{1+r}\right) + \left(\frac{1}{1+\rho}\right)^{t+1} U\left(A_{t+1} + Y_{t+1} - \frac{A_{t+2}}{1+r}\right) + \ldots$$

so that

$$\frac{\partial V}{\partial A_{t+1}} = \left(\frac{1}{1+\rho}\right)^t U'(C_t)\left(\frac{1}{1+r}\right) + \left(\frac{1}{1+\rho}\right)^{t+1} U'(C_{t+1})(-1).$$

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[2]Alternatively, σ measures the aversion of the individual to fluctuations in their consumption over time.

where I have substituted back C_t and C_{t+1} using $C_t = A_t + Y_t - \frac{A_{t+1}}{1+r}$.[3] Setting this expression equal to zero and doing a little algebra yields the condition

$$U'(C_t) = \frac{1+r}{1+\rho} U'(C_{t+1})$$

which you will note is exactly the same as the condition in the two-period model we studied earlier.[4]

D: Another Example

Let the instantaneous utility function be $U(C) = \frac{C^{1-\sigma}-1}{1-\sigma}$ with $\sigma > 0$ as before. The condition for optimal consumption choice is $C_t^{-\sigma} = \frac{1+r}{1+\rho} C_{t+1}^{-\sigma}$ which can be written as $C_{t+1} = \left(\frac{1+r}{1+\rho}\right)^{\frac{1}{\sigma}} C_t$. This condition implies $C_t = \left[\left(\frac{1+r}{1+\rho}\right)^{\frac{1}{\sigma}}\right]^t C_0$. The sequence of constraints on the evolution of assets implies that $A_0 + \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t Y_t = \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t C_t$.[5] In other words, the present value of lifetime consumption is equal to initial wealth, $W_0 = A_0 + \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t Y_t$. This constraint and the optimality condition can be combined to give $W_0 = \sum_{t=0}^{\infty} \left[\left(\frac{1}{1+r}\right)\left(\frac{1+r}{1+\rho}\right)^{\frac{1}{\sigma}}\right]^t C_0$. Now, provided $\left(\frac{1}{1+r}\right)\left(\frac{1+r}{1+\rho}\right)^{\frac{1}{\sigma}} < 1$, as we will assume, the sum converges to $\theta^{-1} = \left[1 - \left(\frac{1}{1+r}\right)\left(\frac{1+r}{1+\rho}\right)^{\frac{1}{\sigma}}\right]^{-1}$ so that we have $C_0 = \theta W_0$ or, more generally, $C_t = \theta W_t$ where $W_t = A_t + \sum_{j=0}^{\infty} \left(\frac{1}{1+r}\right)^j Y_{t+j}$.

The special case of $r = \rho$ is of some interest as then $\theta = \frac{r}{1+r}$ so that the decision rule for consumption can be written as $C_t = \frac{r}{1+r}\left[A_t + \sum_{j=0}^{\infty} \left(\frac{1}{1+r}\right)^j Y_{t+j}\right]$, a quantity that is

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[3]Suppose that we wish to choose x to maximize a function $f(x;a)$ where a is a parameter. The first order condition $\frac{df}{dx} = 0$ implies a solution $x^* = x^*(a)$ which may be substituted into the objective to find the maximized value $f^* = f(x^*(a);a)$. Suppose that we now wish to find how this maximized value changes as the parameter, a, changes. That is, we wish to compute $\frac{df^*}{da}$. The envelope theorem states that $\frac{df^*}{da} = \left.\frac{\partial f}{\partial a}\right|_*$, the partial derivative of $f(x;a)$ with respect to a evaluated at x^*. To prove the theorem consider $\frac{df^*}{da} = \frac{df}{dx}\frac{\partial x^*}{\partial a} + \frac{\partial f}{\partial a} = \left.\frac{\partial f}{\partial a}\right|_*$ as $\frac{df}{dx} = 0$ at the optimum.

[4]The are two other conditions required in the infinite period case. The first, often called the "no Ponzi game" condition is $\lim_{n\to\infty} \left(\frac{1}{1+r}\right)^n A_{t+n} = 0$ which rules out rapidly growing debt. The second is the "transversality condition" which requires $\lim_{n\to\infty} \left(\frac{1}{1+\rho}\right)^n U'(C_{t+n}) A_{t+n} = 0$. This condition rules out consumption paths with low consumption and high accumulation of assets.

[5]Here I have also used the no Ponzi game condition.

sometimes called "permanent income" - the constant rate at which wealth can be consumed forever.

Finally, note that the condition for optimal consumption, $C_t^{-\sigma} = \frac{1+r}{1+\rho} C_{t+1}^{-\sigma}$, can be written as $\frac{C_{t+1}}{C_t} = \left(\frac{1+r}{1+\rho}\right)^{\frac{1}{\sigma}}$ so that defining $\gamma_c = \frac{C_{t+1}}{C_t} - 1$ we have $\gamma_c \simeq \frac{1}{\sigma}(r-\rho)$. We shall make repeated use of this relationship, which holds exactly in continuous time, in our study of economic growth.

E. Stochastic Income

The analysis so far has assumed that the individual knows their future income. Hall [1978] considers the case of an individual with stochastic income. We will assume that even though individuals do not know their future income for certain they can form expectations of that income because they know the distribution of future income conditional on their current information. This is the assumption of rational expectations. As future income is unknown so is future consumption. The optimality condition must be written as $U'(C_t) = \frac{1+r}{1+\rho} E[U'(C_{t+1})|\Omega_t]$ where Ω_t is the information known to the individual in period t. The intuition for the optimality condition developed above holds here provided we replace "marginal utility" with "expected marginal utility" as appropriate.

To better understand the meaning of the optimality condition in this context suppose that $r = \rho$ so that the condition becomes $U'(C_t) = E[U'(C_{t+1})|\Omega_t]$. The idea is that in period t the individual uses all of the information that they have about their future income to make a lifetime consumption plan. They choose current and future levels of consumption so that $E[U'(C_{t+j})|\Omega_t] = U'(C_t)$ for $j = 1, \ldots, \infty$.[6] That is, they satisfy their desire for a "smooth" consumption path by planning a constant marginal utility of consumption and hence a constant level of consumption. Now in period $t+1$ the individual will have more information about their future income (if only because Y_{t+1} becomes known) which, in general, will cause the individual to deviate from the plan made in period t. Thus, $U'(C_{t+1})$ will differed from its planned (or expected) value, $U'(C_t)$, because of the new information that becomes available during period $t+1$. We can write

$$\overbrace{U'(C_{t+1})}^{\text{actual value}} = \overbrace{U'(C_t)}^{\text{planned value}} + \overbrace{\nu_{t+1}}^{\substack{\text{deviation from plan} \\ \text{made in period } t}}$$

where $E[\nu_{t+1}|\Omega_t] = 0$ because ν_{t+1} reflects only new information - that part of Ω_{t+1} not in Ω_t. The condition $E[\nu_{t+1}|\Omega_t] = 0$ implies $E[U'(C_{t+1})|\Omega_t] = E[U'(C_t)|\Omega_t]$ which

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[6]This condition is an immediate consequence of the optimality condition and the law of iterated expectations. Consider the plan made by the individual in period $t+1$. This will have $U'(C_{t+1}) = E[U'(C_{t+2})|\Omega_{t+1}]$ by simply adding one to all of the t's in the optimality condition given above. If we take expectations of both sides of this expression conditional on what is known by the individual in period t, we have $E[U'(C_{t+1})|\Omega_t] = E[E[U'(C_{t+2})|\Omega_{t+1}]|\Omega_t]$. The left-hand side of this expression is just $U'(C_t)$ using the optimality condition in the text. The right-hand side can be shown to equal $E[U'(C_{t+2})|\Omega_t]$. This proves the claim for $j = 2$. We can continue in this way to prove it for all j.

implies $U'(C_t) = E[U'(C_{t+1})|\Omega_t]$ as $E[U'(C_t)|\Omega_t] = U'(C_t)$ because $C_t \in \Omega_t$ by definition.

We can further specialize this model by assuming that the utility function is quadratic so that $U(C) = -\frac{1}{2}(\overline{C} - C)^2$ where $\overline{C}$ is known as the "bliss" level of consumption.[7] In this case, $U'(C) = \overline{C} - C$ so, continuing with the assumption $r = \rho$, the optimality condition can be written $C_t = E[C_{t+1}|\Omega_t]$ which is equivalent to $C_{t+1} = C_t + \mu_{t+1}$ with $E[\mu_{t+1}|\Omega_t] = 0$. All of the explanation given in the previous paragraph applies here if $U'(C_t)$ is replaced with C_t and so on.

The empirical content of the model may be seen by writing the optimality condition as $E[C_{t+1} - C_t|\Omega_t] = 0$ and noting that this implies $E(C_{t+1} - C_t)X_t = 0$ for any $X_t \in \Omega_t$. In other words, nothing known to the individual at time t ought to be useful in predicting the change in consumption so that if we estimate the equation $C_{t+1} - C_t = \alpha + \beta X_t + \zeta_{t+1}$ we should not be able to reject the hypothesis $\beta = 0$.

To find the decision rule for consumption under the assumptions made so far, note that the condition now implies $E[C_{t+j}|\Omega_t] = C_t$ for $j = 1, \ldots, \infty$.[8] Recall that the sequence of constraints can be written as $A_t + \sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j Y_{t+j} = \sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j C_{t+j}$. Taking expectations of both sides conditional on what is known in period t gives

$$A_t + \sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j E[Y_{t+j}|\Omega_t] = \sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j E[C_{t+j}|\Omega_t].$$

Using the optimality condition, the right-hand side of this expression can be written as $\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j C_t = \frac{1+r}{r}C_t$. Substituting this back in and solving for C_t gives the decision rule

$$C_t = \frac{r}{1+r}\left\{A_t + \sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j E[Y_{t+j}|\Omega_t]\right\}.$$

This rule says to consume an amount equal the perpetuity value of expected lifetime wealth.[9] It is the rational expectations version of the permanent income hypothesis.

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[7]We need to impose the condition $E\left[\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j Y_{t+j}\middle|\Omega_t\right] < \overline{C}$ for all t in this case.

[8]See footnote 6.

[9]We can use this decision rule, the law of motion for assets, and a lot of tedious algebra to show that $C_{t+1} - C_t = \frac{r}{1+r}\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j\{E[Y_{t+j+1}|\Omega_{t+1}] - E[Y_{t+j+1}|\Omega_t]\}$ which shows the difference between C_{t+1} and its expected value to be equal to the perperuity value of the present value of the revisions to expected future income due to the information that becomes known in period $t+1$.

F. Stochastic Income and Interest Rates

When both future income and interest rates are unknown the optimality condition is written as $U'(C_t) = E\left[\frac{1+r_{t+1}}{1+\rho}U'(C_{t+1})\Big|\Omega_t\right]$ where r_{t+1} is the real interest rate between periods t and $t+1$. Using the isoelastic utility function this condition can be written as $E\left[\frac{1+r_{t+1}}{1+\rho}\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\Big|\Omega_t\right] = 1$ which is equivalent to $\frac{1+r_{t+1}}{1+\rho}\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} = 1+\eta_{t+1}$ with $E[\eta_{t+1}|\Omega_t] = 0$. Taking logs of this expression gives $log(1+r_{t+1}) - log(1+\rho) - \sigma[log(C_{t+1}) - log(C_t)] = log(1+\eta_{t+1})$. Remembering that $log(1+x) \simeq x$ when x is small and that $log(C_{t+1}) - log(C_t) = \gamma^c_{t+1}$ the growth rate of consumption we can rewrite this expression as $\gamma^c_{t+1} \simeq \alpha + \frac{1}{\sigma}r_{t+1} + \epsilon_{t+1}$ where $\alpha = -\frac{\rho}{\sigma}$ and $\epsilon_{t+1} = -\frac{1}{\sigma}\eta_{t+1}$. This is the equation estimated by Hall [1988] although his derivation is exact and more elegant.[10] It shows how the growth rate of consumption tends to increase when real interest rates are (expected to be) high as consumers respond by postponing consumption and so reduce consumption now relative to that in the future. The size of the response depends on the intertemporal elasticity of substitution, $\frac{1}{\sigma}$, in much the same way the size of a change in demand for a good depends on the elasticity of demand for that good.

Problems:

(1) Verify that the utility function used in the examples satisfies the assumptions in footnote 1.

(2) Suppose that $Y_1 = 10$, $Y_2 = 200$, $A_1 = 50$ $r = .05$ and $U = logC_1 + \frac{1}{.975}logC_2$. Find the optimal amounts of consumption in each period and the corresponging amount of saving in the first period.

(3) Consider the two period model with $U(C) = logC$.

(a) Show that the condition for optimal consumption choice is $C_2 = \left(\frac{1+r}{1+\rho}\right)C_1$.

(b) Show that the optimal choices are $C_1^* = \frac{1+\rho}{2+\rho}W$ and $C_2^* = \frac{1+r}{2+\rho}W$

(c) Show that the response of C_1 to a permanent rise in income exceeds that to a temporary rise in income.

(4) Consider the two period model with $U(C) = logC$ but suppose that we prohibit the individual from borrowing in the first period so that $C_1 \leq A_1 + Y_1$.

(a) Draw and carefully label a diagram showing this individual's budget line.

(b) Show that the optimal first period consumption is given by

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[10] Those of you who have taken Economics 210 may recognize that estimation of this equation by OLS is problematic as r_{t+1} and ϵ_{t+1} will be correlated. Hall is careful to deal with this issue properly.

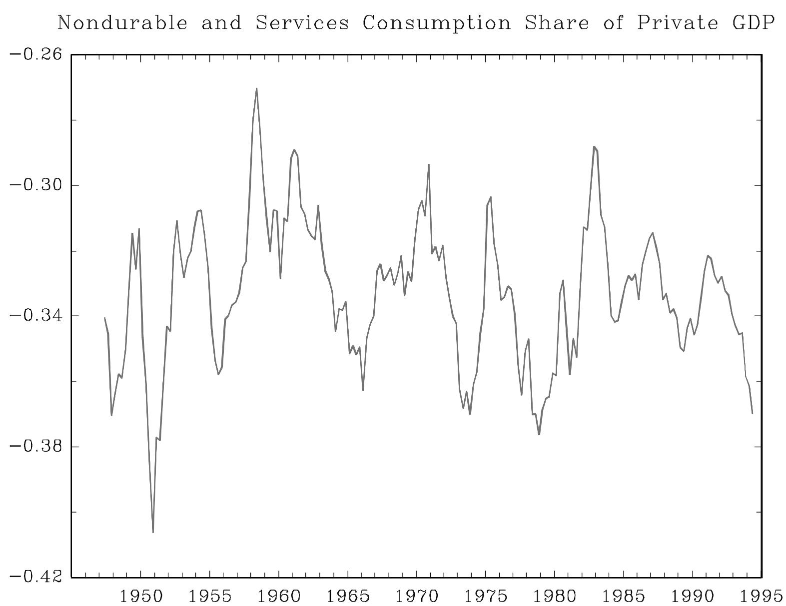
$$C_1 = \begin{cases} A_1 + Y_1 & \text{if } A_1 + Y_1 < \frac{1+\rho}{1+r} Y_2 \\ \frac{1+\rho}{2+\rho}\left(A_1 + Y_1 + \frac{Y_2}{1+r}\right) & \text{if } A_1 + Y_1 \geq \frac{1+\rho}{1+r} Y_2 \end{cases}.$$

HINT: The result in part (b) of problem 3 gives the amount of consumption in the absence of the constraint imposed here. Find the conditions under which the constraint actually restricts the choice made by the individual (we say the constraint "binds" in this case).

(c) On the diagram you drew for part (a) indicate examples of the consumption choices found in part (b).

(d) Compare the response of C_1 to a change in Y_1 when the constraint binds and when it does not.

(5) The chart below shows the log of the share of consumption of nondurable goods and services in private GDP for the US in the post-war period.[11]



[11]Private GDP is GDP less government spending. The concept of consumption in the national accounts (the C in $Y = C + I + G + NX$) differs from that in the analysis in these notes. In the national accounts "consumption" refers to "consumption expenditure" - spending by households on services and both durable and nondurable goods. In the analysis here "consumption" refers to spending by households on services and nondurable goods plus the value of the flow of services from the stock of durable goods. So, the purchase of a new automobile would be included in consumption expenditures but not in consumption in the sense that it is used here. On the other hand, the value of the transportation services provided by the automobile would be included in consumption in the sense that it is used here but not in consumption expenditures. Using the common part of both concepts (spending on services and nondurable goods) is the standard approach. Under certain conditions on the utility function this is without loss and those studies that have used the estimate of the service flow from the stock of durables reach substantially the same conclusions as those who follow the standard practice. Expenditures on durable goods are highly procyclical and best modeled as "investment".

Observe how this ratio rises in recessions and falls in expansions. Is this observation consistent with the theory of consumption discussed in these notes? Why or why not? HINT: Are recessions temporary or permanent?

(6) In this problem we consider the consumption of durable and nondurable goods. The individual chooses $\{C_{t+j}, S_{t+j}\}_{j=0}^{\infty}$ to maximize

$$E_t \sum_{j=0}^{\infty} \left(\frac{1}{1+\rho}\right)^j \left\{ -\frac{1}{2}(\overline{C} - C_{t+j})^2 + \lambda(\overline{C} - C_{t+j})(\overline{K} - K_{t+j}) - \frac{\phi}{2}(\overline{K} - K_{t+j})^2 \right\}$$

subject to

$$A_{t+1} = (1+r)(A_t + y_t - C_t - S_t)$$

$$K_{t+1} = (1-\delta)K_t + S_t$$

$$A_t, K_t \text{ given}, \quad r > 0$$

where all variables have the same meaning as in class except that C_t is now the consumption of nondurable goods. Those introduced here are K_t, the stock of durable goods; $\overline{K}$, the "bliss" level of durables; S_t, spending on new durables; and the parameters, $0 < \delta < 1$, the depreciation rate of durables; $\phi > 0$; and λ. The relative price of durables and nondurables is assumed to be unity for simplicity. Note that "consumption expenditure" as found in the NIPA is $C_t + S_t$.

(a) Explain the terms in the "instantaneous" utility function $U(C,K) = -\frac{1}{2}(\overline{C} - C)^2 + \lambda(\overline{C} - C)(\overline{K} - K) - \frac{\phi}{2}(\overline{K} - K)^2$. What does the sign of λ indicate?

(b) Find the marginal utility of consumption of nondurables.

(c) Show that, if $\rho = r$, then $\Delta C_{t+1} = \lambda \Delta K_{t+1} + \epsilon_{t+1}$ where $E_t \epsilon_{t+1} = 0$. In testing Hall's "random walk model of consumption" some authors have focused on nondurables due to problems in measuring the service flow from and/or stock of durables. How does the result above influence the interpretation of these tests? Under what conditions (both economic as well as mathematical) is this issue not a consideration?