Economics 304
Advanced Topics in Macroeconomics

Notes on Neoclassical Investment Theory

A: The Basic Model

We begin with a model in which firms purchase their capital stock at the beginning of each time period and sell the depreciated capital at the beginning of the next. The important features of the model are (i) investment decisions are reversible; and, (ii) firms take the prices of their output, capital, and labor as given.

At the beginning of period t the firm purchases K_t units of capital at the price P_t^K per unit and hires L_t units of labor at the price W_t per unit. It uses the capital and labor to produce $F(K_t, L_t)$ units of output which it sells at the price P_t per unit. We assume that the production function has the usual properties.[1] The capital depreciates so that $(1-\delta)K_t$ units remain to be sold at the beginning of period $t+1$ when the price of capital is P_{t+1}^K per unit. The firm's nominal profit on all of this is the excess of revenues over costs:

$$\Pi_t = \underbrace{P_t F(K_t, L_t) + \frac{1}{1+i_t} P_{t+1}^K (1-\delta) K_t}_{\text{Revenues}} - \overbrace{(P_t^K K_t + W_t L_t)}^{\text{Costs}}$$

where i_{t+1} is the nominal rate of interest between periods t and $t+1$. The firm's objective is to choose K_t and L_t to maximize Π_t. Before solving that problem, however, note that the second and third terms can be combined as $-P_t^K\left[1 - \frac{(1+\pi_{t+1})(1-\delta)}{1+i_{t+1}}\right]K_t$ where $\pi_{t+1} = \frac{P_{t+1}^K - P_t^K}{P_t^K}$ is the rate of change of the price of capital. Making this substitution in the expression for profits and dividing by the price of output gives the firm's objective as

$$\widehat{\Pi}_t = F(K_t, L_t) - \frac{P_t^K}{P_t}\left[1 - \frac{(1+\pi_{t+1})(1-\delta)}{1+i_{t+1}}\right]K_t - \frac{W_t}{P_t}L_t.$$

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[1]These include: (i) $F(K,0) = 0$ for all K and $F(0,L) = 0$ for all L so that both factors are essential; (ii) $\frac{\partial F}{\partial K} > 0$ and $\frac{\partial F}{\partial L} > 0$ so more of either factor implies more output; and, (iii) $\frac{\partial^2 F}{\partial K^2} \times \frac{\partial^2 F}{\partial L^2} - \left(\frac{\partial^2 F}{\partial K \partial L}\right)^2 < 0$ so that the production function is concave. We often also assume constant returns to scale so that $F(\lambda K, \lambda L) = \lambda F(K, L)$ for all $\lambda > 0$.

The quantity $\frac{P_t^K}{P_t}\left[1-\frac{(1+\pi_{t+1})(1-\delta)}{1+i_{t+1}}\right]$ is known as the real rental price of capital. Note how it plays a role analogous to that played by the real wage, $\frac{W_t}{P_t}$, in determining the cost of producing output.

To solve the problem faced by the firm we have

$$\frac{\partial \widehat{\Pi}_t}{\partial K_t} = \frac{\partial F(K_t, L_t)}{\partial K_t} - \frac{P_t^K}{P_t}\left[1 - \frac{(1+\pi_{t+1})(1-\delta)}{1+i_{t+1}}\right] = 0$$

$$\frac{\partial \widehat{\Pi}_t}{\partial L_t} = \frac{\partial F(K_t, L_t)}{\partial L_t} - \frac{W_t}{P_t} = 0$$

which may be written as

$$\frac{\partial F(K_t, L_t)}{\partial K_t} = \frac{P_t^K}{P_t}\left[1 - \frac{(1+\pi_{t+1})(1-\delta)}{1+i_{t+1}}\right]$$

$$\frac{\partial F(K_t, L_t)}{\partial L_t} = \frac{W_t}{P_t} \qquad .$$

These conditions will be familiar to you as they simply equate the marginal product of each factor of production with the marginal cost of employing the factor. In general these two equations may be solved jointly to find the profit maximizing demands for capital and labor. These conditions, and the assumptions about the shape of the production function, imply that the demand for capital (labor) is a decreasing function of the real rental price (wage). The relationship between the rental price and the interest rate implies the negative relationship between investment and interest rates that is often assumed in more elementary courses.

B. An Example

Suppose the production function is $F(K,L) = AK^\alpha L^{1-\alpha}$ so that $\frac{\partial F}{\partial K} = \alpha K^{\alpha-1}L^{1-\alpha}$ and $\frac{\partial F}{\partial L} = (1-\alpha)K^\alpha L^{-\alpha}$. Letting $\rho_t = \frac{P_t^K}{P_t}\left[1-\frac{(1+\pi_{t+1})(1-\delta)}{1+i_{t+1}}\right]$ be the real rental price of capital and $w_t = \frac{W_t}{P_t}$ be the real wage, the conditions for profit maximization become $\alpha K_t^{\alpha-1}L_t^{1-\alpha} = \rho_t$ and $(1-\alpha)K_t^\alpha L_t^{-\alpha} = w_t$. Taking the ratio of the second of these to the first and doing a little algebra gives $\frac{K_t}{L_t} = \frac{\alpha}{1-\alpha}\frac{w_t}{\rho_t}$ as the optimal capital-labor ratio.

Note that we cannot determine the optimal amounts of capital and labor reflecting the fact that under constant returns to scale and competition, the optimal size of the firm is indeterminant.

C. An Important Special Case

A special case that will be important in our study of economic growth arises when there is only one good in the economy. The good is used for both consumption and investment. In this case, the relative price of capital is always unity so that $P_t^K = P_t$ implying that π_t is the inflation rate. Letting r_t be the real interest rate we have $\frac{1+i_t}{1+\pi_t} = 1 + r_t$ or $r_t \simeq i_t - \pi_t$. The condition for optimal capital stock choice becomes

$$\frac{\partial F(K_t, L_t)}{\partial K_t} = 1 - \frac{1-\delta}{1+r_{t+1}} \simeq r_{t+1} + \delta$$

which may be written as

$$\frac{\partial F(K_t, L_t)}{\partial K_t} - \delta = r_{t+1}.$$

This expression equates the net marginal product of capital, $\frac{\partial F(K_t, L_t)}{\partial K_t} - \delta$, with the real interest rate.